Numerical search for fundamental theory

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History:

Dirac large numbers hypothesis (1937): an observation made by Paul Dirac relating ratios of size scales in the Universe to that of force scales. Dirac noted that the ratio of the size of the visible universe, ct with c the speed of light and t the age of the Universe, to the size of a quantum particle r is about $ct / r = 10^{40}$. Hence, in units $c = 1$ and $r = 1$, this large number can be taken as the age of the Universe, $t = 10^{40}$. There is another ratio with this order of magnitude: the ratio of the electrical to the gravitational forces between a proton and an electron ($\approx 4.4 \times 10^{-40}$).

Eddington number (1938): Arthur Eddington hit on the idea that the fine structure constant α, which had been measured at approximately 1/136, should be exactly 1/136. He based this on aesthetic and numerological arguments. Later measurements placed the value much closer to 1/137, at which point he switched his line of reasoning and claimed that the value should in fact be exactly 1/137, the Eddington number. Wags at the time started calling him "Arthur Adding-one". At this point most other researchers stopped taking his concepts very seriously.

Motivations:

Consider two fundamental constants:

- weak coupling constant g at m_Z
- Weinberg angle Θ_W

In dimensionless units [Tegmark, Aguirre, Ree
Wilczeck (2006)]:

$$g = 0.6520 \pm 0.0001$$
$$\Theta_W = 0.48290 \pm 0.00005$$

Let us choose only three constants: $\pi, e, 1$
and five elementary functions:

$$f_1(x,y) = x + y$$
$$f_2(x,y) = x - y$$
$$f_3(x,y) = xy$$
$$f_4(x,y) = \frac{x}{y}$$
$$f_5(x) = \sqrt{x}$$

We define the complexity of expressions as the total number of elementary functions in the expression. For example,

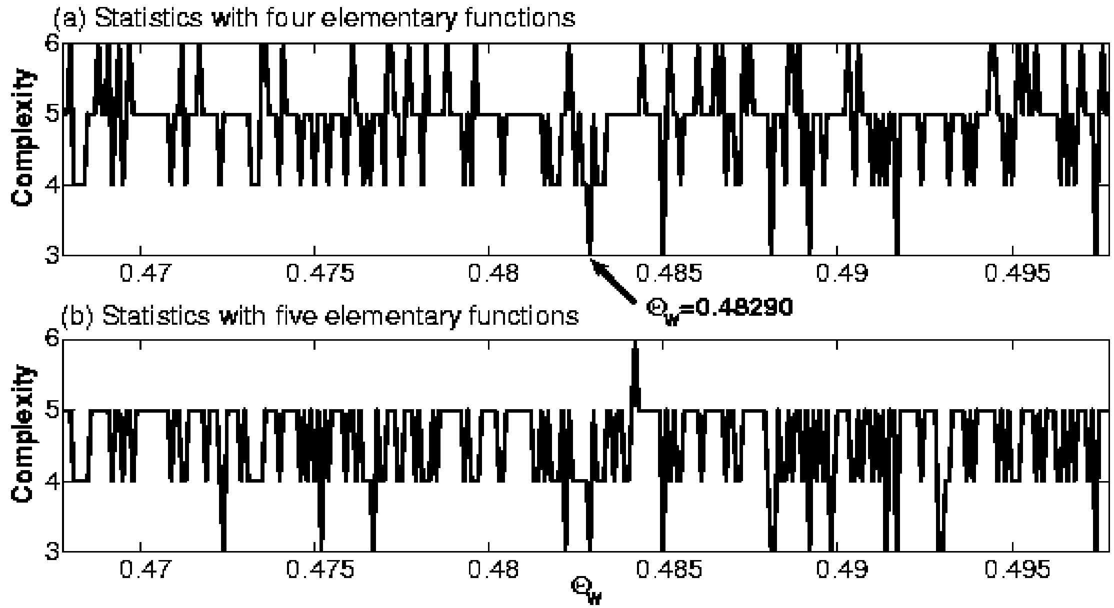
$$M[\frac{2+\pi}{e^2-1}] = M[\frac{(1+1)+\pi}{(e \cdot e)-1}]$$
$$M[f_4(f_1(f_1(1,1),\pi), f_2(f_3(e,e),1))] = 5$$

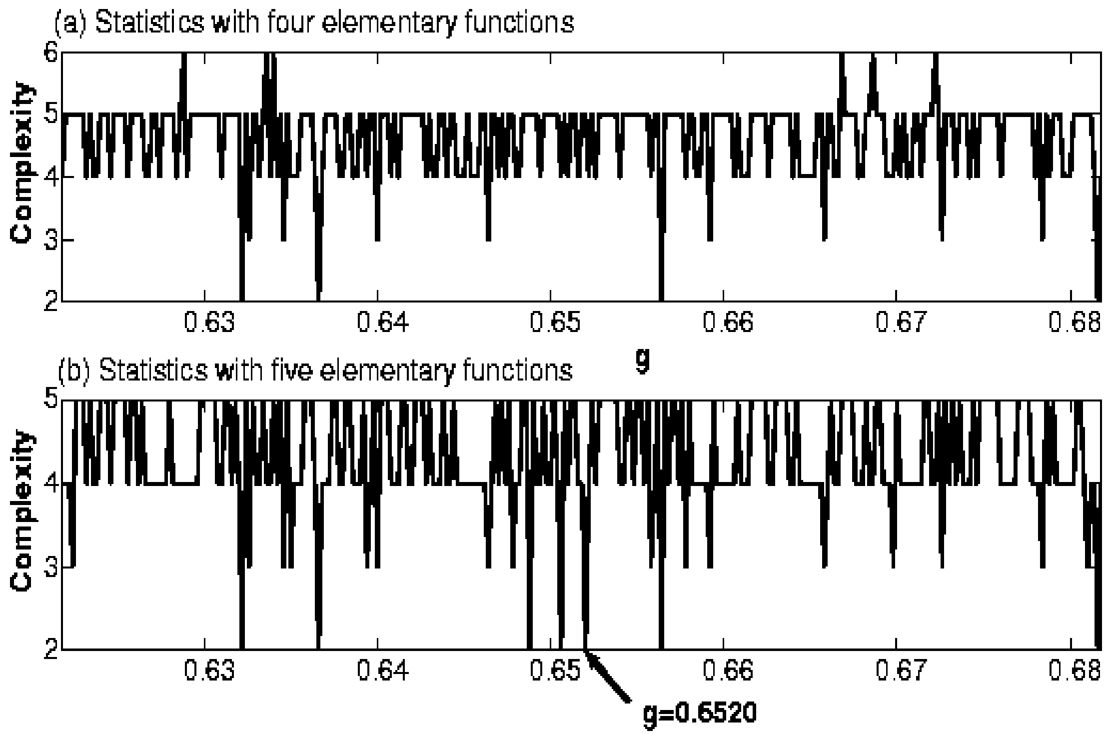
Numerical complexity of an interval is defined the smallest complexity of all expressions that generate a number in the interval.

Discover atypically simple expressions:

$$\Theta_W = \frac{2}{1+\pi} \qquad g = \frac{\sqrt{\pi}}{e}$$

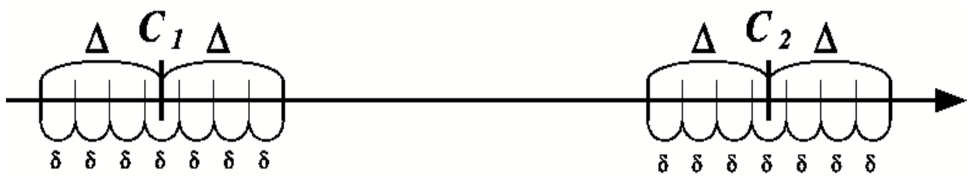
Probability of Θ_W is 1.7 % and of g is 2%





Relation function:

Let F be the set of all function and M be a complexity measure of the set F.



For example, the Kolmogorov complexity is defined as a measure of the computational resource (e.g. the length of the shortest program) needed to specify an object.

Define a two-interval relation function as: $R[I_1, I_2] \equiv min\{M(f)\}$ such that $\exists z \in I_1 \wedge f(z) \in I_2$

Calculate the two-interval relation function $R_0 \equiv R[I_1, I_2]$ for $I_1 = (C_1^m - \delta_1/2, C_1^m + \delta_1/2) \wedge I_2 = (C_2^m - \delta_2/2, C_2^m + \delta_2/2)$

Calculate its expectation value $E \equiv \langle R[I_1, I_2] \rangle$ by averaging over all δ intervals in the neighborhood Δ

Interpretations of the results:

$R_0 \sim E$ nothing new to say => wait for better measurements (smaller δ) of the constants

$R_0 \ll E$ fundamental constants must be related => must study the simple form f to learn new physics

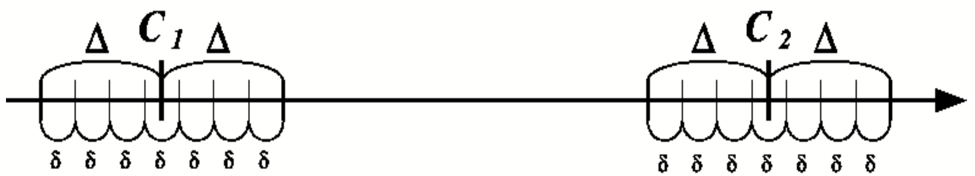
$R_0 \gg E$ both constants are not fundamental => keep looking ...

NOTE: Analysis relies on the definition of the complexity which is not unique. Nevertheless, there are statements that one can make regardless of the exact definition, given that all of the finite expressions have a finite complexity.

In the limit when the size of the interval goes to zero, its complexity grows to infinity. This is true for all of the intervals, unless the interval must contain (due to some fundamental reason) a constant which is given by some fundamental expression. If the expression exists, then for very precise measurements of the fundamental constants it is guaranteed to be discovered.

General analysis:

Define n-interval relation function:



$$R(I_1, I_2, \ldots, I_n) = min\{M(f)\}$$

such that $\exists z_1 \in I_1, z_2 \in I_2 \ldots, z_n \in I_n \wedge f(z_1, z_2, \ldots, z_n) = 0;$

Altogether, there are 2^n- 1 independent relation functions with respect to n precision intervals.

Calculate all of the relation function $R_1, R_2, \ldots, R_{2^n-1}$

Calculate their expectation values $E_1, E_2, \ldots, E_{2^n-1}$

For example $R_{2^n-1} = \frac{1}{N} \sum_{(d1)} \ldots \sum_{(d_n)} R(I_1(d_1), \ldots I_n(d_n))$

where $I_i(d_i) = (x_i + (d_i - \frac{1}{2})\delta, x_i + (d_i + \frac{1}{2})\delta) \subset (x_i - \Delta_i, x_i + \Delta_i)$

For a large number of constants, one might need to know the probability distributions $P_1(R), P_2(R), \ldots, P_{2^n-1}(R)$

The probability of observing a measured value of R_k or smaller: $S_k = \int_0^{R_k} P(R)\, dR$

If $S_k \ll 1$ or $1 - S_k \ll 1$ then the constants in question are atypically related or anti-related.

Relation index: $r = \frac{\sum_{k=1}^{2^n-1} S_k \Theta(\frac{1}{2} - S_k)}{\sum_{k=1}^{2^n-1} \Theta(\frac{1}{2} - S_k)}$ Anti-relation index: $a = \frac{\sum_{k=1}^{2^n-1} (1 - S_k) \Theta(S_k - \frac{1}{2})}{\sum_{k=1}^{2^n-1} \Theta(S_k - \frac{1}{2})}$

Results:

- independent, then $r \sim a \sim 1$
- related, then $r \ll 1$ and $a \sim 1$
- anti-related, then $r \sim 1$ and $a \ll 1$
- related and anti-related, then $r \ll 1$ and $a \ll 1$

Gaussian constants

In a small neighborhood $\vec{\Delta}$ all functions could be considered to be linear, then the sufficient statistics for calculating expectation values could be obtained by varying the value of a single constant $x \equiv C_i$.

Consider all of the function with an equal complexity measure k.

Question: What is the probability distribution $\tilde{P}(x)$ of the range of the functions with complexity k?

The answer depends on the smoothing scale Δ_i .

Problem: Find the average distance from x to the closets $\tilde{x}$ for which $f(C_1^m, C_2^m, \dots, x_0, C_n^m) = 0$ and $M[f] = k$, where $\tilde{x}$ is given by $\tilde{P}_k(\tilde{x})$ and x is given by Gaussian distribution:

$$P(x) = \frac{1}{\delta\sqrt{\pi}} e^{-\left(\frac{x - C_i^m}{\delta}\right)^2}$$

First, let us find the probability distribution for the shortest distance to be s for n generated points:

$$P_1(x) = \tilde{P}(x+s) + \tilde{P}(x-s)$$

$$P_2(x) = 2\left(\tilde{P}(x+s)\int_{x+s}^{\infty} \tilde{P}(x)\,dx + \tilde{P}(x-s)\int_{-\infty}^{x-s} \tilde{P}(x)\,dx\right)$$

$$P_n(x) \sim \left(1 - 2s\tilde{P}(x)\right)^{n-1} n\tilde{P}(x)$$

Let N be the total number of functions with complexity k, then the average distance is

$$\langle x - \tilde{x} \rangle = \int \left(\int (x - \tilde{x})\left(1 - 2(x - \tilde{x})\tilde{P}(x)\right)^{n-1} n\tilde{P}(x)\,d\tilde{x}\right) \frac{1}{\delta\sqrt{\pi}} e^{-\left(\frac{x - C_i^m}{\delta}\right)^2} dx$$

to be compare with its expectation value within its neighborhood Δ_i .

Conclusion

NUMERICS: Weinberg angle and the weak coupling constant have very atypical values.

PHENOMENOLOGY: The proposed analysis can be carried for an arbitrary experiment and might lead to new physics.

THEORY: The procedure provides a numerical technique to test wildly excepted theories and to search for new theories.

FIRST PRINCIPLES: The method allows to derive (and prove) the values of fundamental constants from first principles.

ANTHROPIC PRINCIPLE: Anthropic description of fundamental constants can be explicitly tested.

NUMEROLOGY: Numerical coincidences could be checked for statistical significance.

TODO:

- To develop a better classical algorithm
- To check if quantum computers could help
- To perform new numerical searches