POETRY OF A LOGICAL TRUTH

Paola A. Zizzi

Dipartimento di Matematica Pura ed Applicata
Università di Padova, Via Belzoni, 7
35131 Padova, Italy
zizzi@math.unipd.it

Abstract

We evoke some poetical aspects of a new kind of logical truth (an inner quantum truth) embodying both physical and mathematical concepts, which can be viewed at the same time as a constructive mathematical object, as well as a pure platonic idea.

One day I went to a place where they said the Truth was.
She was not standing, she was sitting, a little bit bent,
Indeed.
Neither young nor beautiful,
Not even smart.
She was "blasè",
As everybody was looking for her,
And when they found her, they were deceived.

Foreword

I would have been delighted to be here with you, and instead in my place there is only a piece of paper, this one. But I hope you will enjoy reading it!
I am a physicist, and, as such, I am in (not requited) love with mathematics and logic.
First of all, I would like to introduce you to a new kind of logical truth, called Truth-Observable, which was born this year in the Mathematical Department of Padova University. It is a mixture of physical and mathematical concepts, and therefore I find it intriguing and somehow poetical.
In Padova, logicians are constructivists, in particular, intuitionists. Then, in a sense, I gave them a gift and a nightmare at the same time, because the Truth-Observable is one of the most constructive mathematical objects (as it arises from a physical theory, more precisely from an inner reversible quantum measurement!) but it is also a logical idea of pure platonic form.
The Truth-Observable is a global truth in the sense that it is conceived as a whole, encoding information from all possible parallel universes. And it is an inner quantum truth, as it is available only from "inside" a closed quantum system.
Finally, the Truth-Observable is subject to a poetical interpretation, as poetry can be considered under both constructive and platonic aspects, is intuited as a whole, and has the power to evocate all possibilities and expectations, like all the possible alternatives in quantum mechanics.
In other words, both poetry and the inner quantum truth deal with superposed classical truth values, but any possible paradox arising from them is such only in a classical sense.

Only one thing is certain, that is,
Nothing is certain.
If this statement is true,
It is also false.

Ancient paradox

Introduction

We know nothing in reality;
For truth lies in an abyss.

Democritus (c. 420 BC)

Poetry and mathematical logic are of course very different. However, there are some analogies between the two, which seem to me quite interesting. Let us start to examine some requirements for poetry and logic separately.
Some requirements for Poetry, dictated by Aesthetics, are:
1) Representation (Poetry incorporates reality, but reconstitutes it accordingly to its own rules).
2) Emotion and Expression (Emotions are felt and shaped in the poem, but they rarely represent perfectly the original feeling. Emotions are not sufficient; significance and depth of thought are also required).
3) Form and Beauty (Internal consistency, coherence, and autonomy).
4) Purpose (Poetry is not only entertainment, pleasure and fun, most poets aim at some altruistic purpose).
5) Psychological and philosophical contents (In general, there are not the very feelings, which are expressed in a poem, but ideas of those feelings, and moreover, the poet generally looks for a catharsis by means of the reader).
Two requirements for **classical** logic, embodied by the *modus ponens* $\frac{A, \quad A \to B}{B}$ are:
1) Topic neutrality (Universal principles, which do not depend on the particular context).
2) Necessity (The principles of logic are derived using only pure reasoning, and do not depend on any contingent features of the physical world).
However, in quantum logic, the *modus ponens* does not hold, so that some versions of quantum logic do not obey topic neutrality (there is dependence on context) and might be contingent in the sense of empirical sciences. This is in fact the case of our inner quantum logic, a paraconsistent symmetric logic, which is sensible to the context, whose axioms are derived by special quantum measurements, and whose logical truth is a generalized quantum observable.
Poetry has its own truth, arising from a pattern of sound arguments, which obey the above mentioned rules/requirements of Aesthetics. Our question is: Does the poetical truth agrees with the logical truth? The answer is negative if the logical truth is classical, because it is exclusionary, while the poetical truth is not. Instead, the answer is affirmative if the logical truth is an inner quantum truth, which is global.

The Truth-Observable

It is not the Truth I am pursuing
But the freedom and happiness
Of its pursuit.

The very word "truth" usually means "logical truth" (while for a physical truth one uses mostly "physical reality"), thus "truth" is in general a logical concept. On the other hand, the word "observable" is a major physical concept. But in an inner quantum world, the "truth" is a logical concept, as well as a physical observable, as it is the truth one "observes" from inside.
The Truth-Observable [1] is a particular kind of generalized quantum observable, resulting in the identity operator of the Hilbert space of the measured quantum system. It is achieved when one interprets a unitary transformation (for example a quantum logic gate in a quantum computer) as a particular case [1] of a generalized quantum measurement [2]. The Truth-Observable can be considered as a global truth, as it is the witness of a superposition. It contains everything with its negation.
The Truth-Observable is a concept pertaining to a very weak logic, namely paraconsistent [3] symmetric logic, which turns out to be the inner quantum logic (or "black box" logic) [4] of a closed quantum system, like a quantum computer.
Paraconsistency, which implies (in addition to the invalidation of the excluded middle principle as in intuitionist logic: $A \vee \neg A$ is false) also the invalidation of the non contradiction principle: $A \wedge \neg A$ is true, which we take as an axiom, as it follows from a particular kind of internal reversible measurement, called "mirror measurement" (or "basic measurement") [1] [4][5], allows superposed states: $A \wedge \neg A$
Finally, the absence of the (usual) cut rule $\dfrac{\Gamma \vdash A \quad A, \Sigma \vdash \Lambda}{\Gamma, \Sigma \vdash \Lambda}$ means that projectors are not allowed, and superposition is never destroyed.
The Truth-Observable can be considered as the inner, global truth of a quantum world.
This is a novel Platonist approach to absolute truth conceived from an internal point of view.
Instead, the Platonist external view of absolute truth is a concept of a stronger logic, namely classical logic.
There is a main difference between the outsider and the insider Platonist views of "absolute" truth. While in the outsider view the absolute truth is an abstract concept, and an "act of faith", in the insider view it can be proven, as it is a generalized observable, and can be measured. In a sense, and quite surprisingly, the inner Platonist view of absolute truth is an extreme product of constructive mathematics!
Notice that a weaker logic means a logic that has fewer structural rules and/or absence of some active contexts. For example, in inner quantum logic, you do not have the weakening and the contraction rules, and there are no active contexts. In a very weak logic, there are many "constraints" in the logical derivation of theorems.
What is the significance of the lack of (usual) sequent calculus in our extremely weak logic?
The external observer cannot recover the quantum "proof" of a theorem step by step. In fact, from a quantum computer, we can only know (by performing an external measurement) whether a theorem

is true or false but the proof will be never accessible to us, as it is lost when quantum parallelism is destroyed by the external measurement.
To be more explicit, let us suppose that theorem "X" can be proven true by both a classical computer and a quantum computer. Also, suppose that an external observer wants to reproduce the proof by a sequent calculus in both cases. In the classical case he succeeds, while in the quantum case he does not, by the previous arguments. So, this is not a usual sequent calculus which can reproduce a quantum proof from outside. Moreover, we think that a usual sequent calculus just cannot exist in inner quantum logic: it should be much more limited than usually, and structured in parallel branches.
A reasonable conclusion might be that usual sequent calculus is just an external logical procedure, that is, the logical reasoning of an observer external to the system about which he is giving judgements.
In particular, if one supposes that theorem "X" can be efficiently proved to be true only by a quantum computer, the fact that the external observer cannot reproduce the proof sounds like a quantum version of Goedel's incompleteness theorem, as was pointed out in a recent work [6].

Mathematical Intuition

In the internal logic of a quantum computer, there are two very strong axioms, the reverse of both the non-contradiction and the excluded middle principles, which are obtained as reversible quantum measurements. These two strong axioms, which are the manifestation of a great amount of quantum information, as the superposed state is maintained, are associated with a very weak calculus (the conclusions are almost similar to the axioms, but the axioms are very strong). The weakness of the sequent calculus indicates that there is almost no algorithm, but the conclusions are not trivial, as the premises are very strong. This might describe the immediacy of mathematical intuition, once the mind is regarded as an internal observer of the whole quantum universe, as in this case the logic it is using is paraconsistent.
It is generally assumed that there is a fundamental difference between axiomatic reasoning and informal mathematical reasoning. Instead, we believe that there are no proofs in mathematics that can be obtained without following the usual path from premises to conclusions, the only difference, in the case of intuition, is that the path is much shorter than usual.
Moreover, since the information stored in the axioms cannot be provided twice as the source cannot be duplicated because of the no-cloning theorem, intuition is given only once. In this sense we agree with Penrose [7], as repeatability is of course an intrinsic feature of algorithms. The loss of repeatability is a property due to the absence of the contraction rule, by which an operation cannot be repeated once the context within a particular sequent has been exhausted.
We think that it is the attitude of the mind to place itself as an internal observer with respect to the quantum computing universe [8] as a whole that allows it to grasp mathematical intuition. In other words, mathematical intuition is an interactive task between the mind and the universe, in some sense different from the understanding of physical laws. Mathematical intuition is a private communication between one single mind and the whole universe [9].
This sounds much like Platonism, but in our case the Platonic world of ideas is replaced by the physical universe. Goedel's first incompleteness theorem, which somehow stresses the imperfectness of the Platonic world, in this context actually reveals the incompleteness of any possible unified physical theory.
Intuitionist logic, for which the law of excluded middle is invalid, replaces Platonism by a constructive approach to mathematics. Surprisingly enough, we see that paraconsistent logic, for which both the laws of non-contradiction and of excluded middle are invalidated, still leaves room for Platonism to a certain degree.

The Self and the Mirror Measurement

In a recent paper [5] we considered a new kind of quantum measurement, performed by a (fictitious) internal observer placed inside a quantum computer on a quantum space background. This measurement is reversible because it is achieved by means of a unitary operator instead of a projector operator. Thus, the superposition can be recovered, and there is no loss of quantum information.
Among all unitary matrices acting on a quantum state as reversible measurements, there is one set of diagonal matrices, that we called "Mirror" measurement, which has the property of leaving the probabilities unchanged, although modifying the probability amplitudes.
The logical consequence of the mirror measurement is that the internal observer gets rid of the non-contradiction principle. By symmetry, he also gets rid of the excluded middle principle. The internal observer is then endowed with a paraconsistent and symmetric logic.
There are two very strong axioms in this logic, the mirror measurement leading to the one which states that: A and not A is true, which is the converse of the non-contradiction principle. The philosophical meaning of this axiom is that a superposed quantum state reflects in a slightly deformed mirror, that is, the diagonal unitary operator "Mirror", which just changes the probabilities amplitudes, but leaves unchanged the truth values (the identity operator being the perfect mirror). This analogy would suggest that a quantum state has undergone a reversible self-measurement that does not destroy the superposition. The act of "looking at itself in the mirror" confirms the objective existence of the quantum state itself.
We believe that the sense of the "Self" in human minds arises when a quantum mental state undergoes a mirror measurement. (Here for a quantum mental state we mean a superposed state of tubulins in the Objective Reduction (OR) model of Penrose-Hameroff [10]). This might sound like the antithesis of consciousness, as any conscious event originates from OR, which is subsequent to a quantum superposition. And in fact, self-awareness should be the purest form of consciousness! The sense of the "Self", however, seems to baffle our beliefs. This is due to the fact that in a mirror measurement there is the maximal conservation of quantum information. And this maximal information is somehow stored and re-used several times, without ever being dispersed in the environment. The sense of the "Self" in fact must be the most inner, persistent, indestructible feature of our mind.

The Universal Self: The Universe and the Mirror

I was just there
A moment ago
Reflecting myself
In the Milky Way.

The universe as a whole can undergo (like any other closed quantum system) a mirror measurement. However, the consequences of this cosmic mirroring are extremely intriguing. In fact, the universe is the ensemble of all existing things, and its mirroring is the mirroring of everything that exists at the same time.
The objective existence of existence itself is recognized.
All things then acquire a collective sense of "Self".

The Universal Self: The Mathematical Truth

One should ignore your beauty
Whole beauty
Lonely beauty
Starring Idea
Tender, lovely Self
Not to lose oneself.

I agree with Deutsch [11] who recognizes the dependence of our mathematical knowledge on Physics. Mathematics is in fact the language by which we express our knowledge of the physical world. The effectiveness of this language is due to the fact that the laws of Physics are Turing-computable functions.
However, like Deutsch, I also believe that our knowledge of the mathematical truth (or mathematical intuition) does not depend on Physics.
When the quantum Universe mirrors itself in a Mirror quantum logic gate, its superposed state gets slightly changed, but maintains the truth values at the "right place". This, as we have seen, is the universal Self. Is not that a kind of universal (although slightly imperfect) mathematical truth?
Actually Brouwer, the founder of Intuitionism, considered the Self closely related to the "Immanent truth" [12].
Also, for Brouwer, "The Ego (Self), at the onset of mathematical activity, is simply given; introspection is its natural form of knowing…the Ego is…Consciousness transformed to Mind. The Primordial Intuition… is direct insight, introspection by and in the individual mind…".
Similarly, we would argue, the Universal "Self" might be related to a "universal immanent truth", which is recognized by the observer as the universal mathematical truth. In other words, when the human mind places itself as an internal observer of the universe as a whole (the master program), it behaves as a fixed point, and is able to grasp the code (the mathematical truth).

The Truth-Observable and mental diseases

Put a sequent here
And an axiom there
You forgot the rule
To build up your mind.

I think that scientists should always keep in mind that they work for the improvement of human mental and physical health, and for the general welfare. And I hope that the inner quantum logic discussed above might be applied to a quantum model of mind, and perhaps even help psychologists and psychiatrists in the cure of mental diseases.
In fact, at the Math Department of Padova, we are trying to launch a European Project aimed at studying the quantum logic of the unconscious.
Of course logic **is not** the psychology of reasoning, as it just tells us how we ought to reason in the correct way. Nevertheless, we found out that logic itself can support the psychology of reasoning.

For example, “mirror measurement” would correspond to a “mirroring” of the “Self", and some anomaly in that process might be responsible for a mental disease like the “borderline”.
The Truth-Observable instead is believed to be strongly related to intuition, particularly to mathematical intuition, but also to schizophrenia.
The communication channel between a mind in a classical state and another mind in a quantum state, corresponding to a (external) quantum measurement, in the framework of standard quantum logic, can be viewed as the old kind of therapy. (A new kind of therapy might require the therapist to reconstruct the paraconsistent logic of the patient from the data).
The bipolar disease is thought to be related to the duality between the “Mirror” and the “Liar” [4] measurements of a quantum mental state. And so on .These topics are under study [13].
So logic (and in particular the logical truth) is surprisingly not only the purest form of mathematical thought, like intuition and creativity, but it is also linked to insanity.
Many great mathematicians suffered from mental diseases (or just mental difficulties). Some examples are: Ludwig Boltzmann (depressed, committed suicide) Cantor (depressed), Goedel (self-starvation) Lie (mental difficulties), Bloch (committed murders, was confined in a psychiatric hospital) Florence Nightingale (bipolar), Emil Post (manic-depressive), and so on.
Ten, statistically at least, it would seem that if you are a great mathematician, you must have some mental disease (but the *vice versa* does not hold!).
Mathematics is like that, after all: incomplete (Goedel), random and pseudo-empirical (Chaitin), creative (Brouwer, Chaitin) and its creative origin is rooted in quantum (unconscious) mental states sometime very close to insanity (Zizzi).
I think that Math cannot be considered anymore as a refuge from everyday life, as in a sense it is even worse!
And also poetry, which brings light on some deep zones of our subconscious, is not easier: beauty and pure thought do not necessarily mean safety, happiness, calm, and rationality…

Who said I am unhappy?
I got my way
In the blue…
Don’t worry
In the blue sky
Oh! I stammer…
In the blue sky
I said…
I see my hope
My Truth
Where it is?
Disaster…

Deduction, Induction, Physics, Logic and Poetry

Even if my theory is false
It is not as false
As your prejudice
About Truth.

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Physics, like all empirical sciences, proceeds by induction from facts (data) to laws and theories, by means of patterns-recognition, brainstorming, creative guessing, and insights. And then, physics goes back from theories and laws to data by deduction, and at the end must agree with experiment. On the other hand, mathematics is a process of pure deductive logic, but does not have to agree with experiment.
Poetry is only inductive.
Much has been said about the beauty of pure Mathematics, maybe too much.
Also, when a physical theory is perfectly embodied in a mathematical theory, they say it gains a higher aesthetic appeal. Instead… when mathematics becomes a little bit more similar to physics, in the sense it has some empirical features, (like in Chaitin's idea of mathematical randomness [14]) they say it becomes less appealing… But Poetry, lyrical beauty, is even more inductive than Physics!!! So we hope that in the future scientists will recognize beauty (and poetry) not only in a physical theory formulated in a correct and precise mathematical way, but also in a mathematical theory which bases its axioms on physical facts. The Truth-Observable is an example of this poetical mixing between logic and physics.

Is the Truth available?

The Truth is available to all
There are many marked paths
But there is an unmarked one
The only way I like to go,
Where the Unknowable
Stands.

There is a difference between the truth of a single theory and the truth of the theory of everything (TOE).
While the first one is, at least in principle knowable, the second one is unknowable, at least in Mathematics, because of Goedel's incompleteness theorem, and Chaitin's mathematical randomness.
Maybe the same holds in Physics, as I personally don't believe that a physical TOE is feasible.
What about the Truth-Observable? As it is a kind of mixture between a physical observable and a logical truth, it should be a challenge for a universal absolute truth. But it is not so.

All we know of the truth
Is that absolute truth, such as it is,
Is beyond our reach.

Nicholas of Cusa (1401-64)
De Docta Ignorantia

Well, in a sense, the Truth-Observable is a global kind of truth, encoding all the quantum information as a whole, but it is not available to an external observer. It is an inner property of the closed quantum system. But if one considers the whole universe as a quantum computer, then the observer is internal, equipped with paraconsistent logic, and can achieve the Truth-Observable

through mathematical intuition. But this is just one single theory, a model universe, and cannot be considered a TOE. Therefore, the Truth-Observable cannot be considered the final, universal truth. In its place, there is the Unknowable, a much more exciting idea!

The gods did not reveal from the beginning
All things to us; but in the course of time
Through seeking, men found that which is better,
But as for certain truth, no man has known it,
Nor will he know it; neither of the gods,
Nor yet of all the things of which I speak.
And even if by chance he were to utter
The final truth, he would himself not know it;
For all is but a woven web of guesses.

Xenophanes (c.570-c. 480 BC) Greek philosopher

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