A Universe without expansion

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We discuss a cosmological model where the universe shrinks rather than expands during the radiation and matter dominated periods. Instead, the Planck mass and all particle masses grow exponentially. Together with a preceding inflationary phase and a late dark energy dominated epoch this model is compatible with all observations. The curvature is almost constant during all epochs. Cosmology has no big bang singularity. There exist other, equivalent choices of field variables for which the universe shows the usual expansion or is static during the radiation or matter dominated epochs. For those "field coordinates" the big bang is singular. Thus the big bang singularity turns out to be related to a singular choice of field coordinates.

After the discovery of general relativity, Einstein and others have tried to find static solutions of cosmology. This attempt has been abandoned after Hubble's observation of a systematic redshift proportional to the distance of a galaxy. This redshift has been taken as a clear indication for the expansion of distances with cosmic time. There is, however, a loophole in the argument. Imagine that masses of electrons and protons were smaller at the time of emission of radiation from a galaxy than they are today. Then the frequencies of characteristic atomic lines are also smaller than the ones observed on earth. This effect could replace the redshift due to expanding distances. In this note we demonstrate that such a scenario is perfectly viable. We construct a simple model for which cosmological distances shrink or remain constant. Only the Planck mass and the particle masses increase simultaneously with time [1].

Our model predicts dynamical dark energy or quintessence [2–9] for late cosmology, while very early cosmology is characterized by an epoch of inflation. Inbetween one finds the usual radiation and matter dominated epochs. For all four periods the absolute value of the Hubble parameter H remains almost constant, given by an intrinsic mass scale μ. While the sign of H is positive for inflation, it turns negative for radiation and matter domination. Nevertheless, we recover all standard predictions of cosmology. Since particle masses grow proportional to the Planck mass all observed bounds on the time variation of fundamental constants and apparent violations of the equivalence principle are obeyed.

The cosmology of our model has no big bang singularity. The field equations admit a solution which can be extended to infinite negative time $t \to -\infty$. In this limit the effective Planck mass and the scale factor approach zero. Invariants formed from the curvature tensor remain finite.

While the general setting of simultaneously varying Planck and particle masses, as well as the Weyl scaling to the equivalent Einstein frame, can be found in ref. [1], the notions of the Universe shrinking during radiation and matter domination are new and related to our specific model. Another striking uncommon property of this model is the almost constant curvature scalar for all epochs. Furthermore, an important feature is the simplicity of our model covering both inflation and present dark energy, dominated by the same simple quadratic potential. Finally, the identification of the big bang singularity as a matter of the choice of field coordinates sheds new light on this old problem.

Field equations. The cosmological field equations can be derived by variation of the effective action Γ which includes already all effects of quantum fluctuations. Our main points can be demonstrated for a simple form of the effective action for a scalar field χ - the cosmon - coupled to gravity,

$$\Gamma = \int d^4x\sqrt{g}\left\{-\frac{1}{2}\chi^2 R + \frac{1}{2}K(\chi)\partial^\mu\chi\partial_\mu\chi + \mu^2\chi^2\right\}. \quad (1)$$

The kinetic term,

$$K(\chi) = \frac{4}{\tilde{\alpha}^2}\frac{m^2}{m^2+\chi^2} + \frac{4}{\alpha^2}\frac{\chi^2}{m^2+\chi^2} - 6, \quad (2)$$

leads to a stable theory for $K > -6$ ($K = -6$ is the "conformal point"). Our choice of $K+6$ interpolates between large values for $\chi^2 \ll m^2$ and a small constant for $\chi^2 \gg m^2$. The specific form of this interpolation does not matter. Compatibility with observations in late cosmology (bounds an early dark energy) requires $\alpha \gtrsim 10$, while a realistic inflationary period in early cosmology can be realized for small $\tilde{\alpha}$, say $\tilde{\alpha} = 10^{-3}$. The present value of χ can be associated with the reduced Planck mass $M = 2.44 \cdot 10^{27}$eV, while the present value of $V = \mu^2\chi^2$ accounts for the dark energy density, such that $\mu \approx 2 \cdot 10^{-33}$eV. Our model differs from a Brans-Dicke theory [10] by three important ingredients: the presence of a potential $V = \mu^2\chi^2$, the χ-dependence of K and, most important, the scaling of all masses with χ [1]. Since μ only sets the scale and can be taken as unity, the only three free parameters of the scalar and gravity part of our model are α,$\tilde{\alpha}$ and m/μ.

For a homogenous and isotropic universe (and for vanishing spatial curvature) the field equations read [1]

$$K(\ddot{\chi} + 3H\dot{\chi}) + \frac{\partial K}{\partial\chi}\dot{\chi}^2 = -\chi(2\mu^2 - R) + q_\chi, \quad (3)$$

$$\begin{aligned} R &= 12H^2 + 6\dot{H} \\ &= 4\mu^2 - (K+6)\frac{\dot{\chi}^2}{\chi^2} - 6\frac{\ddot{\chi}}{\chi} - 18H\frac{\dot{\chi}}{\chi} - \frac{T^\mu_\mu}{\chi^2}. \quad (4)\end{aligned}$$

The constant term $4\mu^2$ on the r.h.s. of eq. (4) corresponds to the potential divided by the squared Planck mass, $4V/\chi^2$. As usual, we denote the scale factor in the Robertson-Walter metric by $a(t)$ and $H = \partial_t \ln a$. The energy momentum tensor $T_{\mu\nu}$ as well as q_χ reflect the effects of matter and radiation.

De Sitter solutions. For constant K eqs. (3),(4) have solutions where the geometry is for all times t a de-Sitter space with constant H, while the effective Planck mass increases exponentially

$$H = b\mu \ , \ \chi = \chi_0 \exp(c\mu t). \tag{5}$$

In the absence of matter ($q_\chi = 0, T^\mu_\mu = 0$) the dimensionless constants b and c obey algebraic equations which have two solutions, determined by

$$\begin{aligned}(K+6)c_1^2 \ &= \ 4 \ , \ (K+6)c_2^2 = \frac{4}{3K+16}, \\ 3bc \ &= \ \frac{2}{K+6} - 2c^2. \end{aligned} \tag{6}$$

The two solutions coincide for $K = -5, c_1 = c_2$.

The solution c_1 exists for all $K > -6$ with $bc = -2/(K+6) < 0$. In this case one has

$$c = \frac{2}{\sqrt{K+6}} \ , \ b = -\frac{1}{\sqrt{K+6}} = -\frac{c}{2}. \tag{7}$$

Furthermore, for $K > -16/3$ one has also the solution c_2 with

$$c = \frac{2}{\sqrt{(K+6)(3K+16)}} \ , \ b = \frac{K+4}{\sqrt{(K+6)(3K+16)}}. \tag{8}$$

Solutions with both b and c positive exist only for $K > -4$. (For $K < -6$ only the solution c_2 is possible, with $bc < 0$).

In order to discriminate between the two solutions we also consider the $0,0$-component of the gravitational field equation

$$3H^2 = \mu^2 + \frac{K}{2}\frac{\dot\chi^2}{\chi^2} - 6H\frac{\dot\chi}{\chi} + \frac{T_{00}}{\chi^2}. \tag{9}$$

In the absence of matter only the solution c_2 (8) is consistent with eq. (9). Thus this solution is the one relevant for scalar field dominated cosmology. The solution c_1 is realized in the presence of radiation, see below.

Asymptotic cosmology. We begin with scalar field dominated cosmology and assume $\tilde\alpha^2 < 2$ such that for $\chi \to 0$ the condition $K > -4$ is obeyed. Then scalar field dominated cosmology describes an exponentially expanding universe with exponentially increasing effective Planck mass χ. As long as constant K remains a good approximation the solution (5), (8) can perfectly describe the evolution of the universe for all times, including $t \to -\infty$. This solution is completely regular, no singularity is encountered. Indeed, we can take for $t \to -\infty, \chi \to 0$ the geometry of a de Sitter space with curvature tensor

$$R_{\mu\nu\rho\sigma} = b^2\mu^2(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}). \tag{10}$$

All invariants formed from the curvature tensor and its covariant derivatives are regular.

The "big bang" is free of any singularities. The central ingredient why the usual singularity is avoided arises from the behavior of the effective Planck mass χ: it approaches zero as $t \to -\infty$. From the point of view of the field equations (3) this is in no way problematic, even though the effective strength of gravity, characterized by the effective Newton-constant $G(\chi) = 1/(8\pi\chi^2)$, diverges for $t \to -\infty$. (A singularity free big bang has been observed in other contexts [11], [12]).

Inflation. We will next show that the first stage of the evolution describes an inflationary universe. Let us take $\tilde\alpha \ll 1$. For the very early universe with $\chi \ll m$ one has $K + 4 = 4/\tilde\alpha^2 - 2 \gg 1$, such that $b \gg c$. In this case we can neglect $\ddot\chi$ as compared to $3H\dot\chi$ in eq. (3). This property is called the "slow roll approximation" for inflation. We may continue the slow roll approximation to larger values of χ. As long as $\chi^2/m^2 \ll \alpha^2/\tilde\alpha^2$ we can neglect in eq. (2) the term $\sim \alpha^{-2}$, such that the evolution equations read in the slow roll approximation

$$H^2 = \frac{\mu^2}{3} \ , \ \dot\chi = \frac{\tilde\alpha^2\mu\chi(m^2+\chi^2)}{\sqrt{3}(m^2 - 3\tilde\alpha^2\chi^2)}. \tag{11}$$

The slow roll approximation breaks down once $\dot\chi/\chi$ is roughly of the same order as H. We may define

$$\tilde\epsilon = \left(\frac{\dot\chi}{H\chi}\right)^2 = \left(\frac{\tilde\alpha^2(m^2+\chi^2)}{m^2 - 3\tilde\alpha^2\chi^2}\right)^2, \tag{12}$$

such that the slow roll period ends once $\tilde\epsilon$ is of the order one. (For large K the criterion is rather $\tilde\epsilon \lesssim K$.) For $\chi^2/m^2 \approx 1/(4\tilde\alpha^2)$ one reaches $\tilde\epsilon \approx 1$ and we conclude that the inflationary slow roll phase ends once χ reaches a value of this order of magnitude. The amplitude of density fluctuations is governed by the ratio of the potential over the fourth power of the effective Planck mass, μ^2/χ^2. For large values of $m^2/(\tilde\alpha^2\mu^2)$ the density fluctuations can be very small, as required for a realistic cosmology.

Radiation domination. After the end of inflation entropy is created and the universe is heated. The subsequent radiation dominated period is realized for large χ where K can be approximated by the constant $4/\alpha^2 - 6$. For radiation the trace of the energy momentum vanishes such that the field equations (3), (4) are not altered by the presence of radiation. However, eq. (9) involves now the energy density of radiation $T_{00} = \rho_r$. Conservation of the energy momentum tensor implies $\rho_r \sim a^{-4}$ and therefore $T_{00} = \bar\rho_r\mu^4 \exp(-4b\mu t)$,

$$\frac{T_{00}}{\chi^2} = \bar\rho_r\mu^2 \exp\big\{-2(c+2b)\mu t\big\}. \tag{13}$$

For the shrinking universe according to the solution (7) the energy density of radiation increases proportional to χ^2, with $T_{00}/\chi^2 = \bar\rho_r\mu^2$. Eq. (9) is then obeyed for

$$\bar\rho_r = -3\frac{K+5}{K+6}. \tag{14}$$

A positive ρ_r requires $K < -5$ or $\alpha^2 > 4$. The scenario of a shrinking radiation dominated universe with increasing effective Planck mass looks rather unfamiliar and intriguing.

We will find out later that this scenario predicts actually the same observations as the standard radiation dominated universe with expanding scale factor and constant Planck mass.

Matter domination. The issue of matter is slightly more complicated. A realistic setting requires that the mass of the nucleon m_n or the electron m_e scale proportional to the growing Planck mass χ. Otherwise the ratio m_n/χ would depend on time, violating the strict observational bounds. (Small deviations from this proportionality are allowed and could result in an observable time variation of fundamental constants and apparent violation of the equivalence principle.) As a consequence of the proportionality of particle masses to χ one finds for massive particles an additional "force" in eq. (3), adding a term $q_\chi = -(\rho - 3p)/\chi$ on the right hand side [1]. Also on the r.h.s of eqs. (4), (9) one has now to add terms $-T^\mu_\mu/\chi^2 = (\rho - 3p)/\chi^2$ and $T_{00}/\chi^2 = \rho/\chi^2$, respectively. For a conserved particle number the density n is diluted as $n \sim a^{-3}$. Thus the energy density of a pressureless gas will scale $\sim \chi a^{-3}$ and therefore become comparable to radiation at some time. After this matter-radiation equality we can essentially neglect radiation and follow the evolution in a matter dominated universe.

For $\rho \sim \chi^2$ (and neglecting p) the additional terms on the r.h.s. of the field equations are constant (after dividing eq. (3) by χ). Solutions of the type (5) are again possible, now with $\rho = \bar{\rho}_m \mu^4 \exp\left\{(-3b + c)\mu t\right\}$,

$$\frac{\rho}{\chi^2} = \bar{\rho}_m \mu^2 \ , \ 3b + c = 0. \tag{15}$$

With the addition of the corresponding terms on the r.h.s eqs. (3), (4), (9) are all obeyed for constant K and

$$c = \sqrt{\frac{2}{K+6}}, \qquad b = -\frac{1}{3}\sqrt{\frac{2}{K+6}} = -\frac{1}{3}c, \tag{16}$$

with

$$\bar{\rho}_m = -\frac{2(3K+14)}{3(K+6)}. \tag{17}$$

This solution exists for $K < -14/3$ or $\alpha^2 > 3$.

At this point cosmology is described by a sequence of three de Sitter geometries, all with exponentially increasing χ. For the first scalar dominated inflationary period the universe is expanding, while it shrinks for the subsequent radiation and matter dominated epochs. The Hubble parameter $H = b\mu$ remains always of the same order of magnitude, changing sign, however, after the end of inflation. Except for short transition periods a numerical solution of eqs. (3), (4), (9) shows very good agreement with the de Sitter solutions if one uses for b and c a time dependent K by inserting $\chi(t)$ in eq. (2).

Dark energy domination. In the present epoch we live in a transition from the matter dominated era to an epoch dominated by dark energy. There are several possible causes for such a transition. First, the kinetial K could be modified such that for the present value of χ it gets large again [13]. For example, K could be periodic in χ. For large K the scaling solution with matter (16), (17) is no longer possible and the universe may return to a scalar field dominated cosmology according to the solution (7). Without a modification of K the mass of some particles as neutrinos could grow faster than χ and become relevant for cosmology in the present epoch. This would be the setting for growing neutrino quintessence [14, 15]. The last alternative is very economical in our context since only the χ-dependence of neutrino masses differs from the one for the charged particles. In particular, the χ-dependence of neutrino masses of the model of ref. [15] introduces only two addictional parameters - the present average neutrino mass and the scale χ_t where neutrino masses grow large. Together with $\alpha, \tilde{\alpha}$ and m/μ this is a rather minimal five-parameter set for an overall description from inflation until now. (Additional "particle physics parameters" determine Ω_m and Ω_b, the relation between temperature and ρ_r or the heating of the Universe after inflation.)

Rescaled coordinates. The interpretation of cosmologies with a variable effective Planck mass becomes more familiar if we choose a different system for the coordinates. We may take a modified time coordinate t' and rescaled scale factor a' according to

$$dt' = \left(\frac{\chi}{M}\right)^\eta dt \ , \ a' = \left(\frac{\chi}{M}\right)^\eta a. \tag{18}$$

This yields a rescaled Hubble parameter

$$H' = \left(\frac{M}{\chi}\right)^\eta \left(H + \eta\frac{\dot{\chi}}{\chi}\right) = \frac{1}{t'}\left(1 + \frac{b}{\eta c}\right), \tag{19}$$

where the last identity uses the scaling solution. For $\eta = 1$ the coordinate t' measures time in units of the inverse dynamical Planck mass χ^{-1}, and similar for the space coordinates. Using t' and a' one finds the usual expansion laws for the radiation and matter dominated epochs. On the other hand, for $\eta = -b/c$ the Hubble parameter vanishes, leading to a static scale factor. For the matter and radiation dominated epochs the choice of coordinates for which the universe is static corresponds to $\eta = 1/3$ or $\eta = 1/2$, respectively. For this choice the geometry is flat Minkowski space. We emphazise, however, that eq. (18) requires the use of a particular solution $\chi(t)$ and should be used only as a demonstration of the importance of the choice of units.

Einstein frame. The proper way of changing units in a field dependent way consists in a field redefinition. This is a change of "field coordinates", not to be confounded with a usual general coordinate transformation. Field transformations change the form of the effective action and the field equations. Nevertheless, the choice of field variables does not matter for physical observables. Performing a Weyl scaling $g_{\mu\nu} = (M^2/\chi^2)g'_{\mu\nu}$ and using for the scalar field the variable

$$\varphi = \frac{2M}{\alpha}\ln\left(\frac{\chi}{\mu}\right), \tag{20}$$

the action (1) reads (omitting primes on $\sqrt{g}, R$ and ∂^μ)

$$\Gamma = \int d^4x\sqrt{g}\left\{-\frac{M^2}{2}R + \frac{k^2}{2}\partial^\mu\varphi\partial_\mu\varphi + M^4\exp\left(-\frac{\alpha\varphi}{M}\right)\right\},$$
$$k^2 = \frac{\alpha^2(K+6)}{4}. \tag{21}$$

Particle masses that scale $\sim\chi$ in the "Jordan frame" (using $g_{\mu\nu}$) are constant in the "Einstein frame" (using $g'_{\mu\nu}$) [1].

For large χ where $k^2 \approx 1$ the effective action (21) describes a standard model for quintessence with an exponential potential. One recovers the known scaling solutions for the radiation ($n = 4$) and matter ($n = 3$) dominated epochs, with a constant fraction of early dark energy $\Omega_h = n/\alpha^2$ [2, 4]. One may verify that the de Sitter solutions (7), (14) and (16), (17) are in one to one correspondence with these scaling solutions. The inflationary period occurs for small or negative φ. It is described by "cosmon inflation" [16] with slow roll parameters $\epsilon = \eta = \tilde{\alpha}^2\chi^2/(2m^2) = 1/(2N)$, with N the number of e-foldings before the end of inflation. For the primordial density fluctuations this leads to a prediction [16] of the spectral index $n = 0.97$ and the scalar to tensor ratio $r = 0.13$. The determination of cosmological parameters by the Planck collaboration [17], $n = 0.96 \pm 0.01$, is consistent with this prediction. The value $r = 0.13$ may be considered borderline, requiring an analysis in the presence of the massive neutrinos and early dark energy of growing quintessence. (Modifications of the potential or kinetial for small χ can realize a smaller value of r [16].) The observed amplitude of density fluctuations measures the parameter combination $\tilde{\alpha}^2\mu^2/m^2 \approx (2/3)\cdot 10^{-10}$, resulting in a second characteristic mass scale $\hat{m} = 2m/\tilde{\alpha} \approx 5\cdot 10^{-28}$eV besides μ. It becomes obvious in the Einstein frame that the cosmology of our model can match with observation.

At this point it may be worthwhile to discuss the origin of the apparent singularity of the big bang in the Einstein frame for the metric where H' diverges $\sim 1/t'$, as given by eq. (19) for $\eta = 1$. For the solutions discussed in this note we can associate the "big bang" with $\chi \to 0$. The curvature tensor formed from the metric $g_{\mu\nu}$ remains finite, cf. eq. (10), while it becomes singular for the metric $g'_{\mu\nu}$ at the time $t' \to 0$ when χ reaches zero. The reason is simply that R' is related to R by a multiplicative factor M^2/χ^2 which diverges for $\chi \to 0$. (The precise relation contains also additive terms involving derivatives of χ.) We conclude that the usual "big bang singularity" is a "coordinate effect" in the space of field variables. There exist simple choices of fields where solutions are regular for all time.

In conclusion, we have constructed a "variable gravity universe" whose main characteristic is a strong time variation of the Planck mass or associated gravitational constant. The masses of atoms or electrons vary proportional to the Planck mass. This can replace the expansion of the universe. A simple model leads to a cosmology with a sequence of inflation, radiation domination, matter domination, dark energy domination which is consistent with present observations. The big bang is free of singularities.

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