Abstract

The original methodology has been presented for determining the relationships between the parameters of the Universe, the whole and its parts. Two basic mathematical constants ($\pi$, $e$) and certain fundamental physical constants have been used for defining significant relations among physical properties. The obtained results are in accordance with the official CODATA values [1]. A simple mathematical representation here shows the power of prediction in physics.

Introduction

The objective of this paper is to show that the connection between physics and mathematics is true, which in line with the contest topic "Trick or Truth: the Mysterious Connection between Physics and Mathematics".

Certain mathematical relations in contemporary physics, which would connect Universe as the whole with some other phenomena, have been presented only in fragments.

Hence, the aim has been to obtain mathematical relations which connect the levels of the structure of matter. This has been achieved by connecting fundamental parameters of the Universe with the basic micro world parameters, using the basic mathematical and physical constants.

The mathematical approach and methodology applied in this paper contain widely accepted and proven postulates, in such a way that old paradoxes are rejected and new ones are not produced. The methodology used here also applies the principles, approaches and methods presented in my essay from the 2013 FQXi contest [2], as well as in its improved version [3].

Basis

Let’s define the relation between the Whole and a part of the mass:

$$m_n = M_n \times 2^{-n}$$  (1)
Where $M_u$ is the mass of the Universe, $m_n$ is the mass of any structure level, the exponent $n$ is any number different for the levels in the hierarchy of matter. For different $n$ (structures), in order to shorten the formulas, one or two letters will be used, for example for proton $p$ instead of $n_p$, or for muon will be $mu$ instead of $n_{mu}$.

Based on (1), the following is true for the proton:

$$p = \log_2 (M_u / m_p)$$  \hspace{1cm} (2)

Similarly, let’s also define the generalized radius:

$$r_n = R_u * 2^{-n/2}$$  \hspace{1cm} (3)

and the time:

$$t_n = \lambda_n / c = h / m_n c^2 = \lambda_p m_p c / m_n c^2 = t_p * 2^{-n-p}$$  \hspace{1cm} (4)

Where: $R_u$ – the radius of the Universe, $r_n$ – the generalized radius [4], $\lambda_n$ – the Compton wavelength, $h$ – the Planck constant and $c$ – the speed of light. Formulas (1), (3) and (4) are the essential for the calculations presented in the Table, which you can see afterwards, just above the Conclusion.

Let’s use the CODATA [1] values of physical constants, with uncertainty in brackets: the inverse fine-structure constant $\alpha$ = 137.035999074(44), the proton-to-electron mass ratio $\mu$ = 1836.15267245(75).

We are using the speed value $c$ = 2.99792458 * 10$^{-8}$ m/s, ($\lambda_p$, $m_p$) in (kg-m-s): the proton mass $m_p$ = 1.672621777 * 10$^{-27}$ kg and the proton Compton wavelength $\lambda_p$ = 1.32140985625 * 10$^{-15}$ m.

The values determined through $\alpha$, $\mu$ are presented with twelve significant digits, while they have been derived through the corrected Koide formula [5] with nine digits, i.e. the number of digits of the input values. Uncertainties of physical quantities obtained through the formulas from the Table, are not presented here, but can be found in my previous works, for example in [10]. Note that notation for exponents in the Table is such that E+53 is actually 10$^{53}$.

Mathematical constants $\pi$, $e$ and values derived from them are presented with 15 significant digits, as that is the limit in used software ($\pi$ = 3.14159265358979, $e$ = 2.71828182845905).

Let’s call the mathematical constant:

$$Cy = e^{2\pi} = 535.491655524765$$  \hspace{1cm} (5)

Let’s assume that the exponent $p$ from (2) is close to the Half Cycle $cy/2$, therefore we can write:

$$p = cy / 2 - \Delta p$$  \hspace{1cm} (6)

Let’s say that $\Delta p$ is the proton shift and if we assume that (7) is true:
\[ \Delta p = 2 - \frac{1}{\mu / \alpha' + 2} = 1.93506094352 \]  

Why relation (7)? Because:
- the proton is considered the originator of matter creation in a narrower sense (substance);
- results obtained are considerably poorer if the value of \( \Delta p \) is changed by a small amount;
- we will show that many simple and worthwhile relations of physical quantities are obtained when:

\[ p = c_y / 2 - \Delta p = 265.810766819 \]  

It is possible to present physical quantities in one of the systems of natural units of measurement [6]. In papers [7] and [8], the applied system of natural units of measurement is defined in such a way that the mass, radius and cycle of the Universe equal 1, so that every structure is expressed as part of the ultimate whole, i.e. a unit, („1“). Only in order for the results to be more visible, they will be presented here in (kg·m·sec):

The mass of the Universe for \( n=p \) from (1) is:

\[ M_u = m_p * 2^p = 1.73944911962 * 10^{+53} \]  

Let’s call the time required to travel the proton Compton wavelength at the speed of light the proton time:

\[ t_p = \lambda_p / c = 4.40774883086 * 10^{-24} \text{ s} \]  

Let’s assume that there is a connection (11a) between the generalized radius \( r_p \) and the Compton proton wavelength \( \lambda_p \) towards the relation between the mass of proton and the mass on the Half Cycle level \( m_{cy/2} \), where:

\[ m_{cy/2} = M_u * 2^{-cy/2} = 1.73944911962 * 10^{+53} * 2^{-267.745827762382} = 4.37407634997 \times 10^{-28} \text{ kg} \]

\[ r_p * m_{cy/2} = \sqrt{2\pi} * \lambda_p * m_p \]

Then:

\[ r_p = \sqrt{2\pi} * \lambda_p * m_p / m_{cy/2} = 1.266598191111 * 10^{-14} \text{ m} \]  

and based on (3) we get:

\[ R_u = r_p * 2^{p/2} = 1.29165299385 * 10^{26} \text{ m} \]  

and the Cycle of the Universe:

\[ T_u = R_u / c = 4.30849062205 * 10^{17} \text{ s} \]
It is evident that the Cycle of the Universe has the time dimensions, while the earlier defined Cycle is a dimensionless quantity.

Alternatively, we can calculate the Cycle of the Universe through the proton time $t_p$:

$$T_u = \sqrt{2\pi} \cdot t_p \cdot 2^{p/2} \cdot m_p / m_{cy/2} = \sqrt{2\pi} \cdot t_p \cdot 2^{cy/2 \cdot p/2} = 4.3084906205 \times 10^{17} \text{s} \quad (13)$$

Let’s define the Planck constant: $h = c \cdot \lambda_p \cdot m_p = 6.62606957321 \times 10^{-34} \text{kg m}^2 \text{s}^{-1} \quad (14)$

and the universal gravitational constant:

$$G = c^2 \cdot R_u / M_u = 6.67383601087 \times 10^{-11} \text{kg}^{-1} \text{m}^3 \text{s}^{-2} \quad (15)$$

CODATA [1] values are: $h=6.62606957(29)\times10^{-34} \text{kg m}^2 \text{s}^{-1}$, $G=6.673 \, 84(80)\times10^{-11} \text{kg}^{-1} \text{m}^3 \text{s}^{-2}$

Now let’s define a large dimensionless number $N$ and call it the number of Planck’s oscillators:

$$N = 2\pi R_u M_u c / h = 6.38707718369 \times 10^{121} \quad (16)$$

Or:

$$N = (2\pi)^{3/2} \cdot 2^{(cy+p)^2/2} = 6.38707718369 \times 10^{121} \quad (17)$$

Application of the number $N$ for calculating parameters $f$ and $q$, as well as certain exponents $n$ can be seen in the Table.

The tau lepton mass is obtained under the assumption that the corrected Koide formula is true, which has been shown in [5].

$$\left( m_{e} + m_{\mu} + m_{\tau} \right) / \left( \sqrt{m_f / \alpha'} + \sqrt{m_{\mu}} + \sqrt{m_{\tau}} \right)^2 - 1 / \pi f = 2 / 3 \quad (18)$$

We have:

Here we can see the connection among the masses of charged particles: electron, muon, tau lepton and proton. That is how we obtained the mass of the third lepton by knowing the first two, plus the proton mass, as the Table shows that $f$ can be expressed through the proton. The CODATA [1] value of the tau lepton mass is $m_{\tau}=3.16747(29) \times 10^{-27} \text{kg}$ while the value obtained in the Table is $m_{\tau}=3.167484976 \times 10^{-27} \text{kg}$, the mass that ideally matches (18).

**Some significant relations**

The Table presents the calculation of relationships between the Whole and its parts and it is an essential methodological innovation in presenting physical quantities, as it provides a well-laid-
out representation of the relations that exist among them. Formulas from (19) to (22) are included in the Table itself.

For each structure level, values for \( r, m \) and \( t \) have been calculated according to the formula in the Table header. Whether it is an elementary, complex or virtual particle, a planet, a star or the Universe as a whole, each level has been assigned a certain mass, radius or time, regardless of whether they have a known physical meaning. The point with the Table is that, besides the values of physical quantities, their mutual relations are important as well. Another advantage is that each significant radius has its corresponding mass (for example, see the classical electron radius in the Table), allowing us to compare the parameters of the same property. If the relation to the defined or previously determined physical quantity is known, the formula can be found in the first column. The values which are underlined are those most commonly found in the literature. The last column contains the description of physical phenomena presented in each level, although it should be noted that one row can contain several significant quantities.

In the Table, structure levels are ordered according to the ascending \( n \).

The Table enables the calculation of \( h, G \) and \( e \) by three properties (\( r, m \) and \( t \)) for each \( n \), i.e. on every level, by equations (23-26):

Planck’s constant:
\[
h = c^2 m_n t_n = 6.62606957321 \times 10^{-34} \text{kgm}^2\text{s}^{-1}
\]  
(23)

The universal gravitational constant:
\[
G = c^2 2^{-n/2} r_n / m_n = 6.67383601087 \times 10^{-11} \text{kg}^{-1}\text{m}^3\text{s}^{-2}
\]  
(24)

Square elementary charge:
\[
e^2 = c^3 m_n t_n / 2\pi\alpha' = 2.30707735308 \times 10^{-28} \text{kgm}^3\text{s}^{-2}
\]  
(25)

Or in statCoulomb:
\[
e = \sqrt{10^9 c^3 m_n t_n / 2\pi\alpha'} = 4.80320450501 \times 10^{-10} \text{statC}
\]  
(26)

The CODATA [1] value for \( e \) is:
\[e=4.80320427(12)\times10^{-10} \text{statC}\]

The paper [10] showed that certain values calculated by using the formulas presented here are significantly more accurate than those determined experimentally.

From the way the Planck mass (\( m_{pl}=m_g \)), length (\( l_{pl}=l_q \)) and time (\( t_{pl}=t_p \)) are calculated in the Table, it is evident that their relation to the Whole is the same, that is:
\[
m_{pl}/M_u = l_{pl}/R_u = t_{pl}/T_u = 2^{-c/4-p/4} * (2\pi)^{-3/4} = 1.25126390816 \times 10^{-61}
\]  
(27)

It can be verified in the Table that the following equation is true:
\[
r_{er}/\lambda_p = \mu / 2\pi\alpha' = 2.13252558501
\]  
(28)
Therefore, the above equation, as well as those presented earlier, are supporting the use of the assumed formulas (7) and (11a). It is especially true that the Table can be employed to determine and use many other relations for the same purpose as (11a).

The formula that is especially significant in the Table is the one for the neutron mass. It has been obtained through the induction method, starting from the idea that if many significant physical quantities in relation to the others contain the fine-structure constant, then it can be expected that the same is true for the neutron. In so doing, all the principles, approaches and methods shown in [3] were used. Occam’s razor [16] played a special role in selecting the simplest solution. The formula is expected to be proven or rebutted through other methods. One of the modifications of the formula was given in (29), where simple relations among key parameters of the Universe can be detected more easily.

\[
\alpha' = \sqrt{\frac{q/(p-ne)-1}{el-p}} = 137.0359990734
\]

(29)

The applied methodology leads to simple relations of the Whole and its parts, i.e. of the key parameters, for example:

\[
\begin{align*}
  m_p m_u m_{cy} m_f^{-3} (2\pi)^{-3} &= 1, & m_{pl}^4 m_p m_{cy} m_f^{-6} (2\pi)^{-3} &= 1, & (h*R_u)^{-4} 2\pi m_u G^2 m_p^5 \lambda_p^6 2^{cy} &= 1, \\
  M_u^{1/3} m_{pl}^{-1/3} r_f^{-1} l_{pl} &= 1, & M_u m_{pl}^{-4} m_f^3 &= 1, & 2\pi h R_u^{-4} M_u G^2 c^{-5} \lambda_p 2^{cy} &= 1, \\
  M_u m_f^{-3} m_q^2 &= 1, & R_u^2 G^2 m_f^6 h^{-4} &= 1
\end{align*}
\]

where \( h = h/2\pi \).

These expressions are very simple, just as is expected for the functioning of nature itself. Many attempts with similar formulas can be found in [11], [12], [13], [14]. In [15, formula 21], it should be: \( R_u^2 G^2 m_f^6 h^{-4} = 1 \).
### Table - Relationships of the Universe and its parts

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>exponent</td>
<td>n = \log_2(M_\nu/m_\nu)</td>
<td>Generalized radius (m)</td>
<td>mass (kg)</td>
<td>time (s)</td>
<td>Note and CODATA values</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Universe</td>
<td>0</td>
<td>(r_n=R_\mu 2^{n/2})</td>
<td>(m_\mu=M_\mu 2^{-n})</td>
<td>(t_n=t_\mu 2^{n-p})</td>
<td>(M_\mu=m_\mu * 2^p), (R_\mu=R_\mu * c)</td>
</tr>
<tr>
<td>(r_\nu=\sqrt{\frac{\hbar}{2\pi m_\nu c}})</td>
<td>(199.667231554)</td>
<td>(1.1452762324E-04)</td>
<td>(1.36751450463E-07)</td>
<td>(5.39105881401E-44)</td>
<td></td>
</tr>
<tr>
<td>(t_\nu=\frac{\hbar}{2\pi m_\nu c})</td>
<td>(202.314227683)</td>
<td>(4.568991183E-05)</td>
<td>(2.1765099345E-08)</td>
<td>(3.8378052103E-43)</td>
<td></td>
</tr>
<tr>
<td>Tau lepton mass (\mu_\nu=6.264.88954)</td>
<td>(1.74299954E-14)</td>
<td>(3.67484898E-27)</td>
<td>(2.3275553753E-24)</td>
<td>(\text{Planck time } t_\nu=5.39106(32) * 10^{-44})</td>
<td></td>
</tr>
<tr>
<td>(m_\mu=3.16747(29) * 10^{-27})kg</td>
<td>(265.808779550)</td>
<td>(1.2674708421E-14)</td>
<td>(1.67492735098E-27)</td>
<td>(4.4016814686E-24)</td>
<td></td>
</tr>
<tr>
<td>Neutron mass (\mu_\nu=6.265.810766819)</td>
<td>(1.2665981911E-14)</td>
<td>(1.6726217700E-27)</td>
<td>(4.40774883087E-24)</td>
<td>(\text{Planck mass } = 2.17651(13) * 10^{-8})m</td>
<td></td>
</tr>
<tr>
<td>(m_\mu=3.1674927351(74) * 10^{-27})kg</td>
<td>(9.10938291(40) * 10^{-24})</td>
<td>(9.4093966958E-23)</td>
<td>(8.9049396958E-23)</td>
<td>(4.4016814686E-24)</td>
<td></td>
</tr>
<tr>
<td>Muon</td>
<td>(268.961366)</td>
<td>(4.250366998E-15)</td>
<td>(1.883533174E-28)</td>
<td>(3.91418821E-23)</td>
<td>(\text{Fundamental mass})</td>
</tr>
<tr>
<td>(m_\mu=8.88024331(22))</td>
<td>(269.752303577)</td>
<td>(3.2313088257E-15)</td>
<td>(1.08862161599E-28)</td>
<td>(6.77232251664E-23)</td>
<td>(\text{Classical electron radius, } r_\text{el}=h/2\pi m_\mu c)</td>
</tr>
<tr>
<td>(r_\text{el}=10938929751(31) * 10^{-31})</td>
<td>(270.147258735)</td>
<td>(2.8179403267E-15)</td>
<td>(8.27910907386E-29)</td>
<td>(8.9049396958E-23)</td>
<td>(8.9049396958E-23)</td>
</tr>
<tr>
<td></td>
<td>(276.653237124)</td>
<td>(2.955841410E-16)</td>
<td>(9.10938929751(31) * 10^{-31})</td>
<td>(8.9049396958E-23)</td>
<td>(8.9049396958E-23)</td>
</tr>
<tr>
<td></td>
<td>(404.628454536)</td>
<td>(4.00161987731E-35)</td>
<td>(2.72338828793E-69)</td>
<td>(2.70710449726E+18)</td>
<td>(2.70710449726E+18)</td>
</tr>
<tr>
<td></td>
<td>(411.726866492)</td>
<td>(1.3806304181E-36)</td>
<td>(1.98735245217E-71)</td>
<td>(3.70970679378E+20)</td>
<td>(3.70970679378E+20)</td>
</tr>
<tr>
<td></td>
<td>(535.491655524765)</td>
<td>(3.2480333854E-55)</td>
<td>(1.09991927168E-108)</td>
<td>(6.70275888024E+57)</td>
<td>(6.70275888024E+57)</td>
</tr>
</tbody>
</table>

*1 From the formula in the first column it can be seen that it is enough to know \(\mu\) and the fine-structure constant to determine the neutron mass. The value and name of the generalized radius in the third column can be seen in [4, page 3].

*2 The fundamental mass has a simple relation (19), see calculations [4, page 3].

*3 \(m_\mu = \frac{h^2}{(T_\mu G_c)} \) (19)

*4 \(r_\text{el} = \frac{h}{2\pi \alpha m_\mu c} \) (20)

*5 \(er\) which is calculated by using the formula for the classical electron radius: \(r_\text{el} = \frac{h}{2\pi \alpha m_\mu c}\)

*6 \(er = 2\log_2(R_\mu / r_\mu) \) (21)

*7 at the \(q\) level there is the generalized radius [10, formula 16], actually the Planck length. The assigned mass is the hypothetical mass quantum [9, Table 1], [4, formula 15], while the time in the fifth column is \(2\pi\) times greater than the Cycle of the Universe. The Planck charge can also be determined here.

*8 level \(e=q+\log_2\alpha\) is responsible for electrical phenomena, and \(e = el + er / 2 = mu + mr / 2 = tu + tr / 2 = 411.726866492\) (22)

where \(mr\) and \(tr\) for the classical muon and the tau lepton radius have been determined in the same way as \(er\) in the formulas (20) and (21).
Conclusion

The claim from the beginning of this paper has been confirmed: **the connection between physics and mathematics is true.** Moreover, there is nothing mysterious in that truth. "Mathematics is ‘reasonably’ effective in fundamental physics. There is ‘pre-established harmony’ between them, because the world is fundamentally mathematical."

The way in which the reality is comprised of elementary structures has been shown in the Table to a certain degree. The physical quantities which have not been included can be determined by the insertion of new rows and the search of relations that they have in the Table. The original spreadsheet Table that enables the relevant calculations, can be sent on request.

The problem of the number of dimensions has not been expressed, and there is no need for it, rather, the matter has been characterized through the mass, radius and time. The value of the radius of the Universe should not be regarded as the radius of the sphere; rather, it is the length which would be traveled at the speed of light over the course of the Cycle of the Universe. It is apparent that this approach has not been hampered by all mentioned the simplifications. On the contrary, the simplifications have led to more accurate results. The method presented in the Table can further produce new accurate values, with consistent application of Occam’s razor [16]. Of course, the knowledge about properties of a physical quantity whose relations are being determined can only help. Therefore, it is possible to obtain new constants as well, which could have further multiple applications.

The formulas have been derived with the use of only seven basic mathematical operations. Mathematical constants \(2\pi, e, \exp(2\pi)\) have been used to a great extent. The approach is rational, encompassing mathematical constants and relations among physical constants in the statement: "The whole and parts are immanently dependent on each other".

Acknowledgement

I would like to thank Dragoslav Stoiljković, PhD, professor of the University of Novi Sad, for his thorough and useful advices.

Novi Sad, February 2015

References: