Fundamental is Non-Random

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Although we use randomness when we don't know any better, a principle of indifference cannot be used to explain anything interesting or fundamental. For example, in thermodynamics it can be shown that the real explanatory work is being done by the Second Law, not the equal *a priori* probability postulate. But to explain the interesting Second Law, many physicists try to retreat to a "random explanation," which fails. Looking at this problem from a different perspective reveals a natural solution: boundary-based explanations that arguably should be viewed as no less fundamental than other physical laws.

I. INTRODUCTION

The question of what is meant by a "fundamental" physical theory is more easily answered in the negative – after all, anyone can dream up a theory that clearly *isn't* fundamental. Suppose some physicists thought they had discovered the ultimate theory, and could boil it down to a few sentences. "The universe picks some rules at random," they might announce, "and it has just randomly happened to pick the very rules that we observe. This explains everything!"

Obviously, no one would hail such a proposal as a breakthrough in fundamental physics. Far from explaining everything, it would explain absolutely nothing. Besides, we already know it's not true. Our best physical theories have revealed beautiful symmetries and mathematical patterns that are at least approximately encoded in the mathematical version of the rules that govern our universe – symmetries that belie any plausible claim of random-rule-generation.

Another group of physicists might try to incorporate these symmetries into a similar claim. "Of all the possible rules that respect these symmetries," they might argue, "our universe has picked some at random, and those are the rules we observe!" Again, not a very impressive claim for a fundamental breakthrough. The next sections will explore why we don't find such explanations satisfying on a fundamental level, but the main reason should be broadly obvious: random explanations are necessarily the *absence* of fundamental explanations. Our most fundamental explanations purport to be *non-random*, to explain "Why this, and not that?". Appeals to randomness just say "Why not?".

This point might hardly seem worth developing into an entire essay. A few string theorists might take a position similar to that of the previous paragraph, but they would be in the minority. And yet *many* physicists, I will argue, have fallen into an essentially similar line of reasoning. Certain aspects of our universe, it is commonly thought, should *only* be explained via randomness – and to the extent that such "random explanations" are not available, it is thought to be a serious problem.

This essay takes the opposing view, arguing that the very concept of a "random explanation" is as meaningless as the above suggestions concerning random laws of physics. Randomness is only a useful rule of thumb if there is nothing fundamental to explain. If there is something fundamental or interesting to explain, randomness cannot possibly do the job.

These are probably 'fighting words' for many people familiar with statistical mechanics, a branch of physics essentially built upon randomness. Its fundamental starting point, after all, is something often called the "equal *a priori* probability postulate": when you don't know any better, all possibilities are equally probable. It is commonly accepted that statistical mechanics explains the laws of thermodynamics, which would seem to be a clear counter-example.

But is this explanation really coming from randomness? The First Law of thermodynamics is essentially just a statement of energy conservation. And we have excellent non-random explanations for

this feature of our universe. (Thanks to Emmy Noether, we know it nicely follows from a time-translation-symmetry.) The essential use of the equal *a priori* probability postulate is to explain the *Second* Law of thermodynamics, the fact that entropy always increases. And, to the eternal concern and seeming bemusement of many physicists, the logical steps from randomness to the Second Law are known to be faulty! They fail without the addition of something to break the time-symmetry, something to single out the future as being different from the past – specifically, the "Past Hypothesis" that entropy was much lower near the Big Bang. [1–3]

In response to this failure, many physicists argue that some other "random explanation" is required to complete the derivation of the Second Law. This essay argues that this is neither possible nor desired. First, we will delve into different types of explanation, where randomness makes sense and where it fails. It works best when aligned with the Second Law, a fact that makes it particularly ill-suited to explaining the Second Law itself. For that, we need the Past Hypothesis: something true about our universe that is essentially the *opposite* of random, clearly pointing us towards another type of fundamental explanation. Following this logic leads to the conclusion that we should take a much closer look at boundary constraints, one of our best non-random explanations, and arguably one of the most fundamental.

II. RANDOMNESS VS. EXPLANATION

Randomness is at its best when your knowledge is at its worst, making it a useful decision-making tool in complex situations. If you believe all lottery numbers are equally likely, you would act rationally to assume a "principle of indifference" when deciding which lottery ticket you should buy. But you could hardly claim that anything about the actual outcome was particularly fundamental. In fact, if there *was* something that made the actual outcome more likely (say, a rigged machine), then the principle of indifference would have led you astray. Randomness can work for us, but only when there's nothing fundamentally interesting that needs explaining.

Whatever one thinks about the validity of "random explanations", it should be obvious that most events can have better, non-random explanations. In classical physics, if you know everything about the current state of the system, you can plug those values into dynamical equations and compute either the future state or an earlier state. Given one state[1], therefore, we can explain other states at different times. When such "dynamical explanations" are available, they're always more fundamental than random explanations. After all, they start with more inputs (and fewer unknowns) and so can always make better predictions.

In practice, dynamical explanations usually don't work as advertised. There's always *something* we don't know, and when those unknowns become important, our predictions are going to be uncertain. You could know the temperature, pressure, and volume of some gas, but that hardly tells you all the details of each molecule. Presented with such a vast number of unknowns, we've found that it's useful to resort to the "equal *a priori* probability postulate" of statistical mechanics. We've found that adding randomness in this manner and then applying the dynamics works remarkably well – we're often able to predict what happens next, even with our lack of knowledge. Viewed in this light, it seems that dynamical and random explanations work together to form an empirically successful package.

But this is simply not a correct reading of the situation. For most dynamical rules, if *everything* is known at some instant, accurate predictions can be made either forward or backward in time. In the partial-uncertainty case, on the other hand, predictions only work properly in the forward time direction. If you try to apply the same logic in reverse, you almost always get the wrong answer (unless you're at thermodynamic equilibrium). Suppose you're trying to use this technique to predict the past of a shattering egg. Even if your knowledge of

[1] We'll circle back to this in due course. Dynamical explanations explain relationships between states, not the states themselves.

the shattering egg was almost complete, you'd still find that the unknown parameters would conspire in unpredictable ways to throw off your dynamical predictions. In general, when analyzing time-reversed movies of physical phenomena, combining dynamic with random explanations fails entirely.

Given this, it should be evident that what is doing the explanatory work in the forward-time case isn't the time-neutral assumption of randomness, but rather something that must necessarily be time-directed. And that something is the Second Law of thermodynamics itself. When the Second Law is in force, there's a nice provable reason why the unknown parameters usually don't matter much. (Of course, sometimes unknowns do matter – an unknown puff of wind can alter a thrown ball – but that's a far cry from air-friction run in time-reverse, where the unknown microscopic details lead to macroscopic effects.) Our empirical success at making predictions from imperfect data is therefore not due to "random explanations", but rather "Second Law explanations". If the randomness were doing the explanatory work, it would operate just as well in reverse.

What really needs explaining, therefore, is the success of the Second Law. It's time-asymmetric, so it can't come about from only our time-symmetric dynamical laws. Indeed, we find that it *supplements* dynamical laws in cases of incomplete knowledge. With this point in mind, we can see why physicists might be tempted to find a "random explanation" of the Second Law, as perhaps the only alternative left standing. The next sections will explore why this doesn't work.

III. ENTROPY AND THE PAST HYPOTHESIS

The Second Law tells us that entropy always increases. So while it is far from maximum today, it must have been even smaller in the past. And indeed our best cosmological observations tell us that the deep past was in a very low entropy state. True, it had typically-high-entropy features like uniform temperature and density, but other features – the smaller-sized universe, the unused free energy that would later result from nuclear fusion and gravitational collapse [3] – make it clear that the entropy of the past was indeed much lower than the entropy of today.

But what is entropy? The relevant parameter here, Boltzmann entropy, is associated with a state of knowledge of the "macrostate" of the system (the big-picture properties), not the actual system itself, which is in some particular "microstate". From what we know about the system (its macrostate-features), we can compute a measure W of the number of different microstates that are compatible with our knowledge. The entropy of the macrostate is engraved on Ludwig Boltzmann's tombstone: $S = k \log W$, where k is fittingly known as Boltzmann's constant.

Note that the entropy is actually associated with a macrostate, a state of inexact knowledge, not a microstate. If we knew the actual state, there would be only one compatible microstate (itself!), and the entropy would be $k \log(1) = 0$. It's only logically possible to talk about assigning entropy to a microstate if there is some clear rule as to what types of macrostate should be considered in the first place.[2] So entropy is a measure of how uncertain you are about which microstate the system is really in. The more possible underlying states, the higher the entropy.

Because of the Second Law (and our inexact knowledge), we can't run dynamics backwards to the Big Bang. So without dynamical explanation, how can we explain the *macrostate* of the early universe? It might seem that one option would be to resort to randomness, to the equal *a priori* probability postulate. If all Big Bang microstates are equally probable, this logic goes, then the Big Bang was overwhelmingly likely to be in a high-entropy macrostate. (Just as any random drop of water is far more likely to be in the Pacific Ocean than in your sink.) Randomness predicts high-entropy.

[2] Unless someone tells you which rule to use (which "coarse-graining"), actual states cannot be said to have any entropy at all!

And yet, we know (from our best observations) that the early universe was clearly a *low* entropy macrostate! Here, the explanation-from-randomness has failed entirely. This is considered by many physicists to be a great and enduring mystery. Alternatively, if one takes the view that random explanations can't possibly explain anything fundamental, then this mismatch is hardly evidence of anything.

One option at this point is just to hypothesize that the Big Bang macrostate was low entropy and take that as a given. Given this "Past Hypothesis", one can easily prove the Second Law. But this is even less informative than a random explanation, the equivalent of the annoying: "Because I said so!". What's more, one can only assign the "low entropy" status to a macrostate, which is a state of knowledge – and any such rule about our knowledge of the early universe could hardly be a fundamental rule. We want to know *why* the early universe had such a smooth distribution of matter – we want to know the *explanation*, and a random explanation doesn't seem to work.

Another option at this point is to use dynamical explanations to explain the Big Bang as a consequence of something in the even-more-distant past, as in the popular "cosmological inflation" models. But as you might imagine, this just shunts the same mystery about the improbable initial state to a different point. As Sean Carroll puts it: "Inflation, therefore, cannot solve this problem all by itself... the initial conditions necessary for getting inflation to start are extremely fine-tuned, more so than those of the conventional Big Bang model it was meant to help fix."[4] Besides, running dynamics *forward* (but not backward) is already in the domain of the Second Law, given imperfect knowledge. Such inflation arguments often use Second-Law-style reasoning when motivating both the onset and the end of inflation, so those arguments could hardly be used to justify the Second Law itself.

So what *might* explain the success of the Second Law? The first person to tackle this problem was Boltzmann himself, after he realized that his "proof" of the Second Law had mistakenly included a time-asymmetric assumption. Boltzmann's instinct then was the same as many physicists today: to forge ahead with "random explanations" anyway! The next section will explore his reasoning, along with some 'Modern Boltzmannian' approaches that are still being pursued.

IV. RANDOM ANTHROPIC EXPLANATIONS

With his statistical understanding of the Second Law, Boltzmann knew it wasn't ironclad. Dynamical processes – with some very low probability – can evolve the actual microstate of the universe into a macrostate with *lower* entropy. If you wait long enough, he reasoned, anything would eventually happen, no matter how improbable. And high-entropy states can't support life and consciousness, so we don't notice the universe until a rare low-entropy moment happens. This is an additional "anthropic explanation" of why we find ourselves in an improbable macrostate: eventually something like our universe would randomly happen, and we find ourselves here because we're not anywhere else.

Before we broach the serious problems with this account, it's worth taking a step back to see what such a "random anthropic explanation" amounts to. The only input requirements are randomness and an infinite amount of time (along with dynamical processes that have a non-zero chance of exploring every point in possibility space). Given these, absolutely anything and everything will eventually happen, and that explains what we see.

This type of story suffers from precisely the same flaws as "random explanations" in general. They can't answer "Why this but not that?", and indeed have to posit "This *and* that." (And how could it be otherwise, with no other starting point or principle?) Such reasoning is the *antithesis* of a fundamental explanation. It's easy to come up with plenty of more-probable options in such a Boltzmann universe – say, a single planet orbiting a single star in a high-entropy background, randomly created at this very moment. (The *most* probable is the "Boltzmann Brain" scenario, where you are some disembodied brain experiencing one blip of consciousness,

before lapsing back into macro-equilbrium.)

Boltzmann's proposal was abandoned, but this general logical thrust – that somehow dynamics and randomness can explain the Second Law – lives on in many other approaches. One recent proposal from Barbour and colleagues [5, 6] notes that essentially *any* group of gravitationally interacting particles will pass through a "Janus Point" where the coarse-grained macrostate is at lowest entropy. If the entropy of the universe is unbounded, the argument goes, entropy will increase in *both* time directions from this special point (which would look like the Big Bang, when rescaled). The Second Law would be due to us being on one side of the Janus Point, for any random history of the universe.

It is easy to see that all the critiques to Boltzmann's proposal apply here as well. In random anthropic reasoning, absolutely anything that can happen, will happen. Furthermore, if a very-coarse-grained Second Law is really coming about from such logic, then it could easily be reversed by the same logic at a finer graining. Taken as a subsystem, the Milky Way and the Andromeda Galaxy are heading for a collision, with its "subsystem Janus point" clearly in the future, not the past – and yet our local Second Law is in disagreement with this argument. For the Second Law to be robust at all scales, it cannot come about randomly.

Another group of Modern Boltzmannians are using a version of cosmological inflation, with a multitude of universes, to try to resolve the improbable-initial-state problem [7, 8]. But almost all of these utilize some type of time-asymmetric/Second-Law-style reasoning. The only hint of a plausible time-neutral solution here would be some variant of a proposal from Carroll and Chen [9]. But even if some serious technical problems [10] are overcome, such an account falls directly into the essential difficulty with random explanations: it would "explain" an infinite number of very different universes, and hence would not really explain anything.

It is my view that these approaches aren't merely unpromising or difficult [11, 12], but rather that they're essentially misguided. Dynamics plus randomness may be popular, but it doesn't actually work without adding in the Second Law from the start. To *explain* the Second Law from something fundamental, we need to understand the smooth matter distribution near the Big Bang, and from a thermodynamic perspective this distribution is essentially *non-random*. Looking to randomness to account for such a situation would be like looking to statistical letter-frequency tables to explain the popularity of George R.R. Martin's novels.

But what other options do we have? Projecting further into the past would only deepen the explanatory mystery. Dynamical explanations can only explain one state in terms of another, lacking a logical starting point. And once we understand that randomness should be off the table, there's really only one other type of physical explanation available. The only reasonable path forward is to think in terms of boundaries.

V. BOUNDARY CONSTRAINTS AS EXPLANATIONS

In classical electromagnetism, the surface of a metallic conductor acts as a boundary constraint on the electric field. Normally these fields can point in any direction, but at the surface of a metal those fields are constrained: they must be aligned perpendicular to the surface. But if applying a principle of indifference to the electric field just outside a metal object, it would be very improbable for all the fields to be perpendicular. "What an amazing coincidence!", a random-explainer might exclaim. "It's so much more organized than I would have expected!"

In this case, at least, we can easily see the explanation. The metal acts as a boundary constraint, which always trumps randomness. In general, physicists only use randomness when we have no other information to go on – but in the case of a boundary condition, we have much better information – making random-logic incorrect and obsolete.

True, one can *also* explain this alignment of the electric field in terms of dynamical rules, electrons moving around in the metal. So for this example, an alternate dynamical explanation is possible. But the crucial point is this: even if the dynamical ex-

planation were not available, the boundary explanation would still go through, and it would explain a scenario that would otherwise seem inexplicably organized.

This same essential argument also applies to the Big Bang; all one needs is a boundary constraint on the universe, and this boundary can naturally explain the smooth character of the early universe. The only essential difference is that the necessary cosmological boundary is one dimension higher than the surface-boundary of a metallic conductor. (3D spatial volumes have 2D boundaries; 4D spacetime-volumes such as our universe have 3D boundaries.) What's more, smoothness and uniformity are completely natural for such boundary constraints, precisely what we we observe. A smooth boundary is really quite simple; a highly-clumped boundary would be far harder to explain.

This is far from a novel idea; after all, the initial state of the universe is often referred to as an "initial boundary condition". The only problem is that many physicists want to then *explain* this boundary condition, via dynamics or randomness. And as we've already seen, neither of those are going to work. Instead, the problem goes away if we simply treat boundary-explanations as fundamental in their own right, framing our physical theories such that the boundaries are just as central as the dynamics.

We use boundaries and boundary constraints all over physics, they're just typically viewed as stand-ins for other explanations rather than being fundamental. We imagine infinite thermal reservoirs, compute the normal modes of laser cavities, and pay special attention to the initial conditions of mechanical systems. Even in our most fundamental physical theories, using some basic Lagrangian density, physicists mathematically fix an external (3D) boundary on every spacetime region of interest.

In most of these cases one could make a case that the boundary condition isn't really fundamental, instead due to dynamics or an earlier state. Even in the Lagrangian case, one could argue that there was a bigger boundary that subsumed the smaller one. But this ignores the clear truth that boundaries can be used to explain systems, in general. And as one expands the size of the system, one approaches the biggest 3D boundary of all – the cosmological boundary of the universe, where the "larger boundary" argument fails. Since we need an ultimate boundary to explain the success of our physical theories, the cosmological boundary must be contributing an essential part of the explanation.

One complaint here might be that the required boundary is unlikely, as viewed from a statistical perspective. But this gets the logical priority of explanation exactly backwards. Consider the case of the metallic conductor: the exact same argument could be made in that case. Someone who knew nothing about the boundary condition would use the random statistics to conclude that all electric field configurations were equally likely. Someone else who knew the boundary condition would have more information, and realize where the random-explainer had gone astray: they used the wrong probability distribution, based on a lack of information. The same argument would go through for the Big Bang; from a boundary-based perspective, the assumption that all microstates are equally likely is just wrong.

A more sophisticated complaint would be that boundary constraints apply to *microstates*, not macrostates – and perfectly smooth microstates are very boring. If the early universe had no perturbations whatsoever, one might guess that the rest of the universe would have no interesting structure. One conventional solution here would be to add "quantum fluctuations" to the initial boundary, but such an approach would violate the very concept of a strict boundary constraint. A better solution, which also works for classical systems, is to note that typical boundaries used in physics only tend to smoothly constrain *half* the parameters on any surface. Even in the example of the metallic conductor, if you consider both electric and magnetic fields, exactly 3 out of the 6 components are constrained at the boundary, with the other 3 components unconstrained. Similarly, when one imposes boundaries in Lagrangian field theory, one imposes a boundary constraint on exactly half the relevant parameters (the field value, but not its normal derivative). This half-constrained information is also a well-known

connection between classical states and quantum uncertainty [13, 14], connecting to the "quantum fluctuation" solution mentioned above.

One last complaint might be from those who just didn't accept that boundary constraints were *ultimately* fundamental, and should in turn have some deeper explanation. And to that, I would have no objection – so long as the deeper explanation was neither dynamical nor random nor anthropic. (Dropping back to one of these modes of explanation is the mistake made far too often.) Such thinking might encourage one to view something like Roger Penrose's "Weyl Curvature Hypothesis" in a more fundamental light.[15] But whether one treats the boundary itself as fundamental, or finds something deeper explaining the boundary in turn, we have finally made it to the point where we can draw a few basic conclusions.

VI. WHAT IS FUNDAMENTAL

The goal of fundamental physics is to find a few simple concepts that can explain everything. One popular concept is the idea of a dynamical equation, which in principle explains one moment in terms of another moment. But this obviously cannot be the whole story, for it's all relational. Explaining the relationship between two things does not really explain either of them. What's needed is some 'starting point'.

Some physicists try to deny any special starting point, and just treat our universe as one possible string of events. In this account, the whole history of our universe could be like the outcome of some lottery machine, with no fundamental explanation as to why things are this way and not some other. But we know how to analyze such situations, using randomness, and it predicts a universe completely at odds with what we actually observe. Our universe is not random after all.

The solution to this dilemma is clear, as outlined in the previous section. At minimum, we need to add a fundamental boundary explanation to the dynamics – the 'starting point' from which the dynamics can finish the explanatory job. The typical form of boundaries in physics is exactly the form that we need at the Big Bang: smooth and boring, at least for half of the parameters in the microstate. If we accept this boundary as a given, we can not only explain what we see, but we can also explain the Second Law of Thermodynamics itself. (And with it, an explanation of why our forward-time predictions are so successful, despite vastly incomplete knowledge.)

Once one is willing to accept boundary explanations as being fundamental, other new perspectives become available. In classical physics, our dynamical equations are often viewed as less fundamental than the boundary-constrained Lagrangian density that generates them. In this "Lagrangian Schema", it's actually the boundary constraint and the Lagrangian density (and a globally extremized action) that are fundamental – dynamical laws are merely a consequence.

There's a subtle but intriguing difference between such a Lagrangian perspective and that of simply adding an initial boundary to classical dynamics. In the Lagrangian case, one puts a boundary around the whole of spacetime, not just in the past. Furthermore, when using a Lagrangian, one only constrains half the parameters on each boundary; the other parameters on the boundary are determined by the solution to the whole problem. A classical dynamics problem, on the other hand, fully specifies the initial boundary but says nothing about the other boundaries. Instead of two ways of looking at the same physics, these are really two competing fundamental accounts.[16] But the main point here is that both of them utilize boundaries as a central explanatory feature. Without fundamental boundaries, all of our explanatory schemas fall apart.

The conclusion is simple: Fundamental boundary explanations need to be taken seriously and literally. Instead of looking for some dynamical explanation of those boundaries – or worse, a random anthropic explanation – we should think about physics that uses boundaries as fundamental ingredients. Our cosmological boundary is as fundamental and non-random as anything we have yet discovered. Only by treating it that way can we move forward in developing even more fundamental explanations of our universe.

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