

The Cosmological Cheshire Cat

Predictable and Unpredictable Dark Matter Properties

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April 23, 2020



Abstract

Facing an under-constrained modelling problem in physics, we often compensate the lack of necessary data by adding model assumptions. In this essay, I show that the under-constrained problem to describe a gravitational lens has multiple different options to yield a self-consistent, well-constrained model. Hence, we obtain several predictions to describe the mass distribution that causes light of a background object to be deflected into observable highly distorted images of this background object. The predictions reproduce these images equally well, but turn out to be otherwise inconsistent with each other. In addition, they claim that all luminous matter content of a gravitational lens is insufficient to cause the observed light deflections. Some “dark” matter is required. By investigating these issues in gravitational lensing as a paramount example of under-constrained problems, I show that there is an alternative to such self-consistent models. It resolves the inconsistencies and greatly reduces the (practical) uncomputabilities in under-constrained problems. Will it also be able to resolve the missing mass problem?

1 Phenomenology of gravitational lensing

“I’ve often seen a cat without a grin,” thought Alice, “but a grin without a cat!” – this exclamation by Lewis Carroll’s protagonist of the well-known novel *Alice in Wonderland*, could have also been uttered by any cosmologist when estimating the mass of the group of galaxies, called the “Smiling Gravitational Lens”, shown in the picture on p. 1. The visible masses of the bright central group-member galaxies, forming the eyes and the nose, do by far not suffice to cause the large deflection of light from a blue background galaxy into several, highly distorted and magnified arcs forming the smile and delineating the face. Hence, this mischievous grin challenges our understanding of matter and its gravitational interaction.

Our theory to describe gravitational lensing is derived from general theory of relativity [11]. Solely based on deductive reasoning from a small set of principles, Einstein stated that light deflection occurs because light follows paths that bend in the vicinity of massive objects. If the mass density of the deflecting object exceeds a certain threshold, light can pass by the object via several curved paths, such that multiple, highly distorted images of the source are observed. The effect is called strong gravitational lensing and is detailed in Fig. 1.

Believing that Einstein’s theory correctly predicts light deflection, as it did for solar light deflection [3], we face a lack of mass for extragalactical gravitational lenses. Alternatively, we can deny the universality of physical laws and argue that nature need not be uniform. Then, we have to modify general relativity on extragalactical scales to reconcile theory and observations. Both approaches aim at a radical change in our world view, predicting the existence of new particles or modifying gravity. This drastic step may be avoided by reviewing the methodological ansatz leading us to this challenging conclusion. Thus, we need to search the gravitational lensing formalism for oversimplified assumptions which implies a missing mass. In addition, it is important to identify assumptions leading to predictions that are unrealistic to be ever corroborated or refuted by observations because gravitational lensing is limited to observations without the possibility to experiment. In the next sections, we will see that such assumptions inflate the parameter space to describe gravitational lenses. They are also responsible for inconsistencies because different models with different assumptions can be equally well compatible with the constraining data but can yield contradicting predictions about the same lens [10].

In the following, when investigating the gravitational lensing formalism, we will adopt the viewpoint that we are searching for a missing mass rather than a modification of gravity. So far, many approaches to modify gravity have been proven to be inconsistent with observational evidence like [5, 9]. This does not imply that attempts to modify gravity are futile. Recent work to systematically set up feasible modifications of gravity [12] rather reveals that these can be constrained by the forms and dynamics of the gravitationally interacting matter.

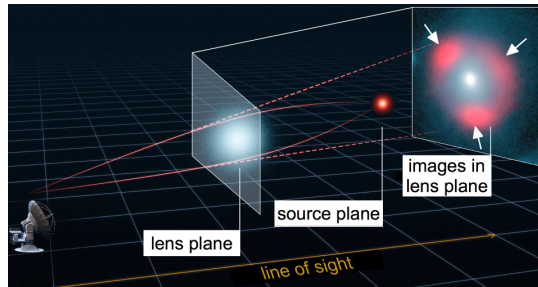


Figure 1: Gravitational lensing: light emitted from a source object in the source plane travels towards us and is deflected by a heavy mass located along its way in the lens plane. As a consequence, we do not observe the light bundles as emitted from the source. Instead, we see highly distorted and (de-)magnified images of the source around the light-deflecting mass which occludes the source object. If the mass exceeds a certain density, the light travels via several paths to us, such that multiple images become observable.

2 Self-consistent lens modelling to predict dark matter distributions

As shown in Fig. 1, the distorted images of the source object are the only observables of this light-deflecting phenomenon. The source is always occluded by the lens and the morphology of the gravitational lens is not fully known if it contains an unknown amount and distribution of dark matter. Further occlusion effects, limits on the brightness of observable objects, and the fact that our observations are two-dimensional projections onto the sky impede us from tracking the paths of deflected light as predicted by general relativity. It does not seem reasonable to model the light paths in detail by means of general relativity because we are unlikely to obtain testable predictions about these paths along the line of sight. Therefore, as a first simplifying assumption to overcome the under-constrained model of light deflection along multiple paths through a three-dimensional mass distribution, we set up an effective description for gravitational lensing. We assume that all deflecting masses between the light-emitting source and the observer are projected into one plane orthogonal to the line of sight where the most massive light deflector judged by its luminous contents is located. Consequently, all results and predictions deduced from the effective gravitational lensing formalism depend on this simplification.

Yet, the number and distribution of multiple images is still too sparse to reconstruct the deflecting mass density in the lens plane and infer the integrated mass of the deflecting objects. Without knowing the source morphology and brightness, it is impossible to reconstruct the gravitational lens which causes the distortions and magnifications observed in the multiple images. Vice versa, we are not able to reconstruct the common source object because the lens may contain unobservable matter. The standard way to resolve this under-constrained problem is to compensate the lack of constraining data by adding physically motivated prior assumptions. These are introduced in the form of so called lens models, i.e. models of projected mass distributions that represent the total mass density distribution of the lens, including a dark matter part, if existing. As summarised in Fig. 2, the iterative process of adjusting the parameters of a lens model by a common source reconstruction and, in turn, optimising the source reconstruction with updated parameters of the lens model, yields a self-consistent picture of a gravitational lens and the respective reconstruction of the deflected background object.

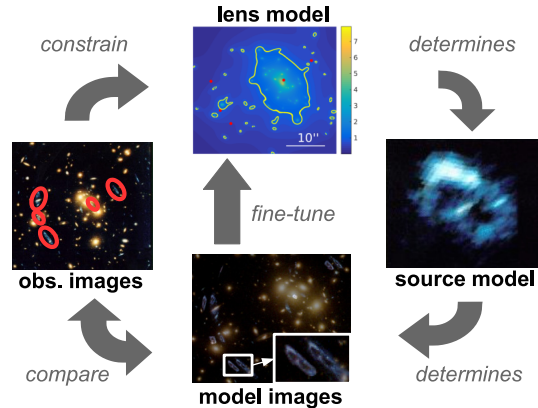


Figure 2: Using a lens model to back-project the observable brightness profiles of multiple images to the source plane, the model parameters are determined such that the back-projected images have maximum overlap. The maximally overlapping back-projections then form a first reconstruction of the source object. This source reconstruction is propagated forward through the lens model into the lens plane and its model-based multiple images are compared to the observed ones. If their coincidence is sufficient, consistent lens and source reconstructions are obtained. If not, the lens model is fine-tuned and the back-projection and forward propagation are iterated until the model matches the observation to the desired degree of precision.

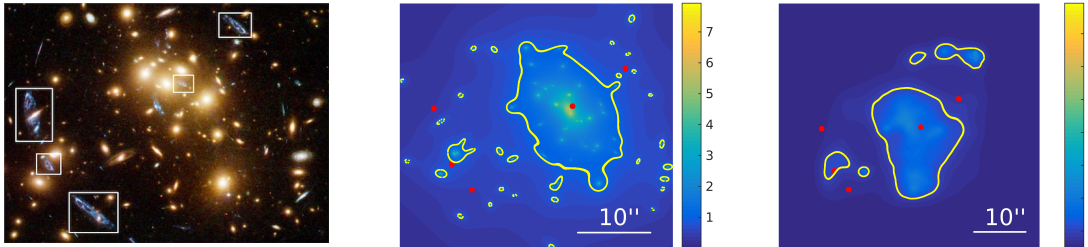


Figure 3: Examples of lens reconstructions [14]: observations of the galaxy cluster CL0024 reveal several systems of multiple images of galaxies behind CL0024 with one prominent five-image-system of a blue source galaxy (left). All images serve as constraints for two different reconstructions of a normalised mass density distribution of CL0024 imposing different assumptions about the lens (centre, right). While both normalised mass density maps cannot simultaneously describe CL0024, the locations of the five multiple images of the blue galaxy (red points) are equally well explained in both models.

3 Inconsistencies between self-consistent lens models

Usually employed lens models are either specific, parametrised mass density profiles or sets of flexible basis functions with adjustable weights, e.g. [6], or [7]. Examples of reconstructed projected mass density distributions for a galaxy-cluster-scale gravitational lens using the algorithms of [6] and [7] are shown in Fig. 3. The projected mass density is normalised to a quantity without physical units called “convergence”, κ . Comparing the κ -maps, it is obvious that employing a parametric lens model and assuming that the luminous matter content traces the dark one, as used by [6], leads to a different result than the basis-function-based lens model of [7], which requires that the mass density model does not cause multiple images to appear at positions where none were observed. Since both lens models can approximately equally well explain the locations of the multiple images, the explanatory power is no criterion for model selection. Neither is Occam’s razor because the complexity of both models is approximately equal in terms of adjustable parameters and parameter ranges and both agree that a dark matter component is required. Popper’s suggested hierarchy to prefer the model which is most easiest to falsify could be used. It may favour the model of [7] because future observations could reveal multiple images in regions that the model assumed to be image-free. But this assumption may be irrelevant in the regions where it just has been disproven, so that the explanatory power of the model with slightly changed prerequisites leading to the same κ -map remained the same.

Hence, comparing gravitational lens models with different assumed prior knowledge, cases occur for which we cannot refute models on the basis of their ability to reconstruct the observational evidence. Even ranking models by the falsifiability of the underlying assumptions is difficult, not knowing the relevance of these assumptions. We also rely on the luck of finding a suitable observation instead of constructing an experiment. For the assumption that luminous matter traces dark matter, it seems unrealistic to have an observational probe soon. The latter could not have been established in the last forty years of observable strong gravitational lensing effects and would require an independent detection and tracking of the dark matter content.

Despite these inconsistencies, it has been shown that a multitude of different lens models usually agree to the integrated mass of deflecting objects within gravitational lenses [8]. Early-day lens reconstructions employed simple lens models of high symmetry, like circular models, and required a large amount of dark matter to explain the positions and shapes of the multiple images. Systematically decreasing the symmetry of the lens models, [1] discovered that the necessary amount of dark matter can be reduced to explain the same image configurations. Yet, with approximately 80% of missing mass in a lens like CL0024, even highly asymmetric lens models cannot avoid a dark matter component to fit the observational constraints.

4 Discarding unknown unknowns in lens reconstructions

As shown in Fig. 3, the approach to reconstruct the deflecting mass distribution by means of a self-consistent lens and source modelling, leads to mutually contradicting models whose verisimilitude may not be ranked relative to each other or refuted by observational evidence. Apart from these fundamental limits of predicting power, the high dimensional, extended model parameter space needs to be sampled to find a solution for the non-linear optimisation problem that underlies the algorithm shown in Fig. 2. Thus, determining a self-consistent model is computationally very intensive and the outcome may not even be the globally optimal solution.

Luckily, it is possible to resolve the inconsistencies and reduce the computational complexity to a minimum at the same time. To do so, we analyse which properties of gravitational lenses are evidence-based and agreed upon by all models. The gravitational lensing formalism only makes point-wise statements. At best they are also valid for a neighbourhood of a point, in which the lens or source properties can be approximated as being constant. For points, at which evidence in form of observable multiple images is present, it is clear that local lens properties can be calculated from their information content. With increasing distance from these locations, our evidence-based knowledge fades and lens model assumptions dominate the prediction of the local lens properties there, as sketched in Fig. 4.

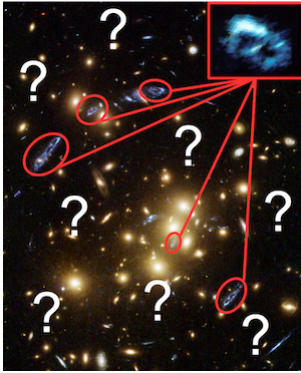


Figure 4: In [13] the lensing region on the sky is partitioned into areas of extended multiple images and areas without multiple images. With the observables from extended multiple images (in the red ellipses), local lens properties are directly determined and from those, their source is reconstructed (in the red rectangle). In the remaining areas, no information from multiple images is available, so that any reconstruction of their mass density distribution solely relies on inter- and extrapolations from the local lens properties relying on model assumptions.

We thus abandon the self-consistent closure and the goal to determine the entire mass density in the lens plane and restrict our lens reconstruction to the locations in the lens plane that show observable multiple images. Practically, as further detailed in [13], the local lens properties are determined by mapping the multiple images onto each other, i.e. without the self-consistent back-projection onto the source. Correlating the multiple images with each other in this way enables us to characterise the lensing effects that are different between the multiple image locations. This is the maximum information about a lens all lens models agree upon because this is the information that is directly obtainable from observables, without imposing a lens model. As a consequence, the inconsistencies are resolved in this new approach. Since it does neither require a lens model, nor back-projections, the respective mathematical optimisation problem is simplified such that the global optimum is always attained within less than a second. It yields the three retrievable local lens properties per image for the current data quality.

If desired, the local lens properties as obtained from this approach can now serve to rank lens-model-based lens reconstructions according to their agreement with the lens-model-independent properties. For the case shown in Fig. 3, we find a high and approximately equal degree of agreement for the lens reconstructions of [6] and [7]. From this, we can conclude that the agreement is most likely a consequence of the same underlying lensing formalism which, by construction, dominantly constrains the local lens properties in lens-model-based reconstructions as well. Thus, the result that the lens-model-independent reconstruction also hints at a missing mass component implies that oversimplified lens models are not the reason for the missing mass.

5 Knowledge gain and remaining darkness

A thorough investigation of the predictions that gravitational lensing can make about the deflecting mass distribution has revealed that predictions of originally under-constrained models have to be taken with great care. The predictive power depends on the approach by which means the under-constrained model is turned into a well-posed problem. On the one hand, this can be achieved by self-consistency, i.e. compensating the insufficient data by additional assumptions that do not contradict each other nor are already in tension with the observational evidence. On the other hand, we also obtain a well-posed problem by separating the under-constrained problem into a constrainable part without causing tensions to the observational evidence and a part that cannot be constrained with the available evidence, which we neglect further on. Developing an entire observation-based cosmology in this way has already been pursued in [4], which inspired the integration of gravitational lensing into this framework.

The self-consistent closure yields a prediction with a strong model dependency, such that it is easy to refute with suitable additional evidence. Yet, unfortunately, the predictions may also go far into the regime where evidence is unlikely to be gained, leaving the prediction there untestable. Different, contradicting predictions could thus co-exist forever, both claiming to describe the same reality. Beyond that, incorporating new evidence into the model to tighten the prediction or refute underlying assumptions is a computationally intensive endeavour.

The second ansatz yields a prediction in the fashion as inductivism tries to set up a universal law: The method relies mainly on the observational evidence with only a small amount of assumptions and the prediction is iteratively extended by adding new data. Characterising the self-consistent closure as a top-down model rejection concept, this ansatz can be seen as a bottom-up model assembly concept. While the latter suffers from the lack of predictive power compared to the former, it has the advantage that, with increasing evidence, it increases our knowledge. Falsifying a prediction of a self-consistent model, we have only eliminated one model. A large amount of others still has to be tested and, potentially, an entire set of degenerate equally viable predictions will remain.

This also seems to be the state of knowledge that we have about dark matter. We falsified the hypothesis that oversimplified lens models in the lens plane are the reasons why dark matter is required to describe gravitational lensing effects. It might turn out, that a model with a three-dimensional deflecting mass does not require any dark matter to explain the future observational evidence that could become available to constrain such an extended mass profile. But until we prove that some dark particles, shown to exist in future direct detection experiments, are the main constituents of gravitational lenses, both self-consistent dark matter models will be in agreement with the observations.

So the grin without a cat still keeps on grinning and we can watch it with the same words as Alice in Wonderland: “It’s the most curious thing I ever saw in my life!”.

Acknowledgements

I cordially thank all my collaborators, colleagues, friends, and everybody who contributed to the progress of this research project, in particular Rüdiger Vaas and Kai König for their advice and support. I gratefully acknowledge funding by the Deutsche Forschungsgemeinschaft (DFG) WA3547/1-3.

Image credits: NASA/ESA (p. 1); ALMA (ESO/NRAO/NAOJ), L. Calçada (ESO), Y. Hezaveh et al. (p. 2); [2] (p. 3); [14] (p. 4); NASA, ESA, M. J. Jee, Johns Hopkins University, [2] (p. 5)

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