FQXi's 2017-2018 Essay Contest
**'What Is "Fundamental"'**

# Indra's Net
## (Holomorphic Fundamentalness)

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**Abstract.** If "fundamental" means something that is at the root of everything, then the physical laws and the objects to which they apply seem to be fundamental. But by looking at the mathematical structure of various theories in physics, we see that "fundamentalness" is relative, revealing a holistic nature. Various types of holism also appear in quantum theory, in Bohm's idea of implicate order, and in the holographic principle. This essay goes beyond these, by proposing a type of fundamentalness as a mathematically consistent basis for these forms of holism, the physical laws, and the ontology of physics. The discussion is based on various examples from particle physics and its mathematical formulation, and implications to what is "fundamental" are analyzed.

## ☰ Searching for the fundamental

The universe is rich in complex phenomena and situations of infinite diversity, yet somehow we seem to be able to understand it to some degree, at least partially, in terms of a small number of laws and concepts. Apparently, we can do this because there is much redundancy in how the world is.

There are more different ways to analyze the world in terms of fundamental constituents. This depends on our interests, which can be the nature of the substances from which things are made, how they work, their purpose, the units of knowledge in which we translate the world when we picture it inside our minds, or even what various aspects of the world make us feel. And for each of these ways there are many paths we can follow to reach an answer. This makes the quest extremely difficult, especially if we don't have a clear idea of what we mean by "fundamental". And it is not easy to define it, given that reality tends to ignore our definitions.

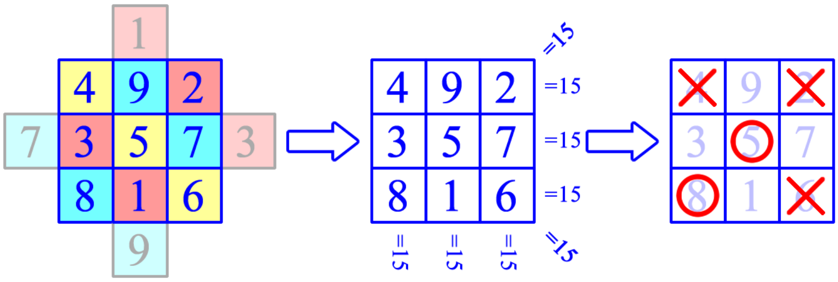
In the following, I will focus on a particular meaning of "fundamental" as something that is at the root of everything. We will analyze examples, mainly from physics, to understand the elusive concept of *fundamentalness*.

## ☱ Isomorphic stories

Let's begin by playing a game called *number scrabble*. Two players take turns to pick one integer from 1 to 9, and each number can be picked only once. The winner is the one who collects 3 numbers totalizing 15, otherwise the game is draw. This game is known to be *isomorphic* to another game, which is much more visual.

$$1\quad \boxed{2}\quad 3\quad \boxed{4}\quad 5\quad \boxed{6}\quad 7\quad 8\quad 9$$

This table allows the kid to play number scrabble indirectly, by playing *tic-tac-toe*, for which there is a simple strategy to never lose.




For the person playing number scrabble the elements of the game, its board, and the rules are completely different than for the one playing tic-tac-toe, who doesn't even think at numbers.

In the following, isomorphisms of mathematical structures will play a central role.

## ☵ Quantum holism

Could the world be made out of a small number of fundamental building blocks, by putting them together like we do with bricks when building a house? Our experience of simpler objects combining into more complex ones makes the idea that matter is made of atoms so appealing to us. Atoms also seem to be made of more fundamental particles, but in a completely different way from our naive idea of bricks. The behavior of particles is expressed by *Schrödinger*-like relativistic wave equations. Atoms, molecules, even larger objects, are described by such waves. Sometimes the *wavefunction* spreads and interferes, but if you catch it, you catch the entire particle, not only that part of its wavefunction you thought was there. This wave spreads everywhere, can be split by sending it through some slits or a beam splitter, but in some sense it is undivided.

A single particle is a wave of various possible shapes. If a particle can have two possible shapes, it can also have any *superposition* or *linear combination* of these shapes. So all possible shapes of a wavefunction form a *vector space*. A property of a particle, like position or momentum, is well defined only for some of the shapes. For example no shape can have well defined position and momentum simultaneously. But the shapes which don't have a given property are superpositions of shapes having it. When you observe a particular property of a particle, the shape of its wavefunction is found to be one which has that property, even if you thought from previous observations that the shape was different. Any shape is a linear combination of other shapes, so in some sense, it contains them all, and it can turn out to be any of them by looking for the right property.

For the two electrons in the Helium atom even this image of waves spreading in space is wrong, since the two electrons are not separate waves. They are in some sense a single wave on a higher dimensional space, depending not only on the $(x, y, z)$ coordinates, but on two copies of them. When we put together two or more particles, the resulting wavefunction is no longer a shape on space, but it is a tensor product of such spaces, and the total state is separable only if it is a tensor product of single particle shapes. The particles in an atom, molecules, and various larger systems, are not separate waves, they are superpositions of tensor products of separate waves. Particles don't have separate identities, but you can still catch one as if it has. This way of composing particles to get larger systems differs from our usual experience even more, because there are infinitely many ways to express quantum systems as superpositions of systems made of separable waves. Can we still say that the wavefunction of a single particle is something fundamental?

It is even more difficult to understand particles as individual when they are created or annihilated. The best way we know to describe this is to see them not as independent parts, but as excitations of a vacuum quantum field. Is this field fundamental, or the excitations are? Or maybe both? These excitations are also in superpositions like the ones described earlier. Moreover, for an observer in accelerated motion, or on curved spacetime, the vacuum itself appears as being excited, that is, as containing particles, even if for another observer the vacuum seems to be empty[1]. The number of particles is not even something well defined like an integer, and depends on the observer.

As we shall see, mathematics offer more adequate notions of composability and reducibility than our classical intuition does, and has something important to tell us about fundamentalness.

# ☷ Relativity of fundamentalness

In mathematics, isomorphisms are ubiquitous. A mathematical structure can admit different but equivalent descriptions or formulations. For example, Euclidean geometry can be formulated axiomatically, as Euclid and Hilbert did (Hilbert, 2013), but it can also be formulated in terms of *symmetries*, as Klein did in his *Erlangen Program* (Klein, 1893), or in terms of numbers and equations, formulation called *analytic geometry*. Also distinct theories may turn out to be equivalent, for example logical operations with propositions are equivalent to operations with sets, and also to logical gates.

To see what kinds of mathematical structures are useful in particle physics and what they tell us about fundamentalness, we start with the Euclidean plane. Hilbert axiomatized Euclidean geometry in a way which distinguishes various mathematical structures associated. Some of the axioms refer to the relations between points and lines, ignoring distances and angles. What results is *affine geometry*. Now one may wonder, why ignoring the angles and the distances? The reason is that they are not uniquely determined by the relations between lines and points. If you pick any two segments lying on distinct intersecting lines, there is a unique *inner product* or *metric* according to which these segments are perpendicular and their lengths as units. This uniquely determines the distance between any two points and the angle between any two lines. There are infinitely many such choices.

The Euclidean plane can be seen as a hierarchy of mathematical structures: it is a set, a topological space, a metric space (where distances are defined), a differential manifold, an affine space, and if we choose an origin, a vector space. Can we say that one of these structures is more fundamental than the others? For example, the topological structure can be obtained from the metric structure, but it also exists in affine geometry, where no metric is chosen. Metric spaces and topological spaces don't always admit a differential structure or even a definite dimension.

Moreover, in the plane affine geometry axioms we can swap "points" with "lines", "collinear" and "concurrent" *etc*, and we get the same axioms. This isomorphism with itself (*automorphism*) leads to *projective geometry*.

So which object is more fundamental? We can regard points as more fundamental, lines being just sets of points, or we can regard lines as more fundamental, points being the meeting points of lines. We can even axiomatize geometry in terms of completely different structures, like circles, in which case the axioms of Euclid or Hilbert become theorems.

We can reduce plane geometry to numbers, using coordinates. Points are identified by pairs of real numbers, while lines, circles, ellipses, and other curves are described by equations to be satisfied by the coordinates of their points. Are the pairs of numbers and the equations more fundamental? If we change the reference frame the coordinates change too, so these numbers can't be fundamental.

But we can ignore the different forms in which we express various objects in different coordinates, and focus on coordinate-independent *invariants*. For example, when changing the coordinates, a line remains a line, and a point remains a point. The invariants are usually equivalence classes of the different coordinate representations of the same object. So why not define geometry as the study of the structures that are invariant to various transformations? This insight was proposed by Klein in the *Erlangen program*. Thus, the inhomogeneous linear transformations of the plane define the affine geometry, the homogeneous linear transformation define the vector geometry, the isometries define the Euclidean geometry, the projective transformations the projective geometry *etc.* In this way, what is fundamental are the symmetries and their invariants. We can still use coordinates, in which case the various transformations can be expressed in terms of matrix multiplication.

Symmetry transformations are fundamental in physics. In special relativity, the *principle of relativity* says that the physical laws and the scalar, vector, a and tensor fields are invariants of the Poincaré symmetry. In gauge theory, various interactions can be understood as arising from other symmetries. Symmetries also explain conservation laws, as shown by Emmy Noether.

The relativity of fundamentalness implied by different axiomatizations and formulations is just *epistemological fundamentalness*. But the examples from the quantum world seem to imply that reality is holistic and there is a relativity of the ontology itself. Should we then take the whole universe as ontologically fundamental? Should we consider that what is fundamental are not the various sets of principles from which everything can be derived, but rather an equivalence class of them? Or maybe it is possible that something more fundamental than these exists?

## ☴ Holographic fundamentalness

For Christiaan Huygens, waves behave as if each point of spacetime becomes a new source. In quantum mechanics a similar picture is given by the *propagator*. The propagator was used by Feynman in his *path integral formalism*, in which all possible histories contribute to reality.

David Bohm saw in the propagator an order present everywhere, *unfolding* and *enfolding* continuously, alternating between being *implicate* and *explicate* (Bohm and Hiley, 1993; Bohm, 1995). He used as a metaphor two concentric cylinders, the space between them being filled with a liquid having high viscosity, like glycerin. Drops of ink are placed in the glycerin at various times, while the interior cylinder is slowly rotated. The rotation spreads the ink droplets into the glycerin, making them disappear, but undoing the rotation makes them reappear, in a succession which gives the impression of particles in motion. He made this metaphor rigorous in an alternative formulation of quantum mechanics which could be used to explain the order which unfolds and enfolds while the wavefunction propagates. He applied this formulation both to standard quantum mechanics and to his own interpretation. Bohm compared his holistic vision with a hologram, where every part encodes information about the whole.

Another application of holography is the *holographic principle*, proposed in cosmology ('t Hooft, 1993; Susskind, 1995), motivated by the observation that a black hole can contain a maximum amount of information proportional to the area of its horizon (Bekenstein, 1973; Bardeen et al., 1973). The idea is that the information in a region is encoded on the boundary of that region. It was further developed in the *gauge-gravity duality* conjecture (Maldacena, 1999), which proposes that there is a correspondence between *quantum gravity* and a *conformal field theory* on the conformal boundary of spacetime. These proposals are not yet proven.

Next, I propose a kind of holism that goes way beyond the ones mentioned so far.

# ☵ Seeds for universes

To understand the idea of holism which I propose here, it is useful to discuss first a much simpler example, which contains many of its features.

Let $f$ be a function defined on a domain of the complex plane $\mathbb{C}$, valued in $\mathbb{C}$, so that its derivatives at any point $z_0$ are independent on the direction in the complex plane in which we vary $z$. This is equivalent to $f$ being *holomorphic*, which means that it can be expanded in power series of $z$ at $z_0$, so it is also analytic. It is also equivalent to the condition that $f$ depends only on $z$, not on its conjugate $\overline{z}$ [2]. Complex holomorphic functions have very nice properties, such as the following holographic property: by knowing its values on a contour, you can find its values everywhere inside the contour. Holomorphic functions are *conformal* – they preserve the angles, but not necessarily the lengths. A complex function $f$ is holomorphic iff it satisfies the *Cauchy-Riemann* equations.

The power series at $z_0$ converges on a disk centered around $z_0$, but you can move to another point of the disk and write the power series at that point, and obtain another convergence disk. By this analytic continuation you can extend $f$ to its maximal domain. So we can fully recover a holomorphic function just by knowing its derivatives of all orders at a single point $z_0$. The collection of the derivatives at a point form a *germ*, in the sense of "seed". This holistic property is much stronger than just obtaining $f$ from some boundary conditions, since in this case *the complete information about the hole is contained in every point.*

If the function $f$ has isolated singularities where it can't be analytically continued, and outside of them is holomorphic, it is called *meromorphic*. Meromorphic functions can be used to represent the electric field in two dimensions, assuming that the charges are localized at points. Then charges correspond to singularities. This suggests the possibility of describing the Maxwell equations using some generalization of complex meromorphic functions.

The first idea to generalize complex holomorphic functions to a 4-dimensional spacetime may be to use *quaternions*, since $1, i, j, k$ span a 4-dimensional space. Indeed we can cast Maxwell's equations in terms of quaternions, but quaternions are not Lorentz invariant. To make this generalization we need Clifford or *geometric algebras*.

If $(V, \cdot)$ is a vector space with an inner product, then its geometric algebra is the associative algebra defined by extending the product

$$ab := a \cdot b + a \wedge b, \tag{1}$$

where $a, b \in V$ and $\wedge$ denotes the exterior product. If $V$ is real and the matrix form of the inner product, when diagonalized, has on its diagonal $r$ pluses and $s$ minuses, then the geometric algebra is denoted by $C\ell_{r,s}$. An orthonormal basis of $V$ gives a basis for $C\ell_{r,s}$ made of all products of subsets of the basis of $V$, so the dimension of $C\ell_{r,s}$ is $2^{r+s}$. The elements of $C\ell_{r,s}$ are the same as those of the exterior algebra $\bigwedge^\bullet V$, but the Clifford product is associative and includes both the exterior product and the inner product. If $V$ is of complex dimension $n$, then its geometric algebra is denoted by $\mathbb{C}\ell_n$, and its dimension is $2^n$.

Coming back to the complex plane, we can obtain it using the geometric algebra $C\ell_{0,2}$ as follows. Take a basis $(j, k)$ of the 2-dimensional real vector space endowed with a negative definite inner product $\cdot$, so $j \cdot j = k \cdot k = -1$, and define the associative product for which $jk = -kj$ and $j^2 = k^2 = -1$. Then, $i := jk = j \wedge k$ rotates $j$ into $k$, since $ij = k$, and gives a complex structure on the plane. We see that in this case $i$ is not intrinsically imaginary, it is just a rotation of the real 2-dimensional plane spanned by $j$ and $k$. The notation $i, j, k$ is not accidental, since $C\ell_{0,2}$ is isomorphic with the algebra of quaternions $\mathbb{H}$. The Cauchy-Riemann equations become

$$\left(j\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right) f = 0. \tag{2}$$

The differential operator $j\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}$ is a generalization of the operator introduced by Dirac to write the equation of the electron (Dirac, P.A.M., 1928), and is called the *Dirac operator*.

The geometric algebra of the Lorentz metric, which can be diagonalized to $(+,-,-,-)$, is the *spacetime algebra* $C\ell_{1,3}$. The Dirac operator appears in the equation of the electron, but it can also be used to write the Maxwell equations in a single equation.

The geometric product is very compact, since it encodes many apparently distinct operations like the inner product, the exterior product, scalar multiplication, multiplications between vectors and scalars, and in 3 dimensions the vector product, in a single associative product. Also, the Dirac operator includes differential operators like gradient, divergence, curl (in 3 dimensions). Its square is, depending on the signature, the Laplacian or the d'Allembertian appearing in the wave equation. In fact, much of the mathematical physics can be formulated in terms of geometric algebras (Hestenes and Sobczyk, 2012).

Geometric algebras allow us to express in a compact way many equations in physics. What we say in this formalism can also be said in the usual mathematical language, but in a less compact form. This is not accidental – it is due to the fact that geometric algebra includes apparently different parts of mathematical physics in an essentially more fundamental way. Moreover, spacetime algebra naturally includes the *spin group* $\mathrm{Spin}_{1,3}$. Its leads naturally to fermions, which are *spinors*. Spinors and tensors belong to representations of the spin group. In addition, the *Fock spaces* of states of many fermions, including the entangled ones, as well as important operators like those of *creation* and *annihilation*, are naturally represented using geometric algebras.

The history of physics showed us that what we considered as fundamental will be replaced by something more fundamental. This usually comes together with unifications. For example, special relativity unified space and time, but also energy and momentum, and the electric and magnetic fields into a single 2-form. If there are fundamental geometric structures in physics, we expect them to bring not only a unification of the formalism, but also of principles and of entities like particles and fields. If we expect that the holomorphic fundamentalness plays a role in physics, probably the way is by geometric algebras[3]. And we shall see in the next section that a unified model of the particles based on a geometric algebra is possible, suggesting that fundamentalness may be holomorphic.

## ☶ Towards a holomorphic unification

The *Standard Model* of particle physics contains the *electroweak* and *color* forces, three generations of *leptons* and *quarks*, their antiparticles, and the *Higgs boson*. The types of particles and their properties are known mainly from experiments. It is believed that more fundamental principles and structures yet to be known determine all these particles, their properties, and other parameters of the Standard Model taken from experiments. And we partially know something about these more fundamental reasons – when combining special relativity with the unitary symmetry of quantum mechanics it follows that particles have to belong to representations of the spin group $\mathrm{Spin}_{1,3}$ combined with translations, which explains their spins and also the equations they follow (Wigner, 1939). Particles are also representations of the gauge symmetry groups, resulting in the electroweak and color interactions.

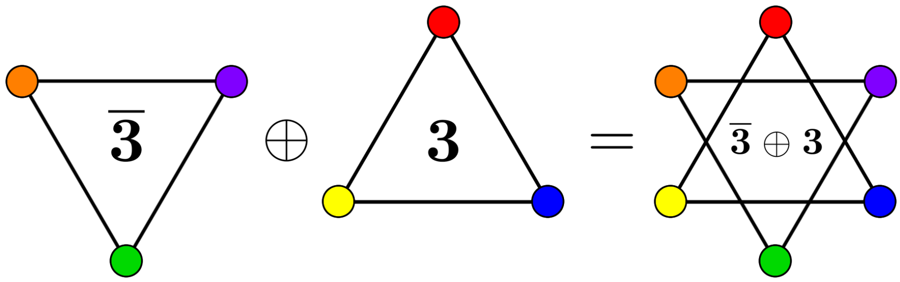
A major question is *why these particular gauge symmetries and representations?*

After the unification of the electromagnetic and weak interactions in a larger gauge group, it was believed that there is an even larger group which also contains the color gauge symmetry. Such proposals are based on the Lie groups SU(5) (Georgi and Glashow, 1974) and Spin(10) (Georgi, 1975; Fritzsch and Minkowski, 1975). Unfortunately, unifications based on a larger gauge group predict new interaction bosons, which would lead to proton decay, contradicting the experiments. Also they didn't explain why these particular representations out of infinitely many possible for each group.

Fortunately, there are other options. Rather than unifying the gauge groups in a larger group, we can unify their Lie algebras in a larger algebra which is not necessary a Lie algebra. It can be a geometric algebra, as I will briefly explain in the following. Technical details can be found in (Stoica, 2017b)[4].

The color degrees of freedom live in a complex 3-dimensional vector space denoted by $\mathbf{3}$. The color gauge group is the group of linear transformations of $\mathbf{3}$ preserving a Hermitian inner product, whose determinant is 1. The electromagnetic gauge symmetry can be represented on $\mathbf{3}$ as the complex phase, so a vector of $\mathbf{3}$ has a color and electric charge, equal by definition with $-\frac{1}{3}$. The exterior algebra $\bigwedge^\bullet\mathbf{3}$ of $\mathbf{3}$ has $2^3 = 8$ complex dimensions. The various exterior powers $\bigwedge^k\mathbf{3}$ are spanned by exterior products of the elements of a basis $(\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3)$ of $\mathbf{3}$, this leading to a basis of $\bigwedge^\bullet\mathbf{3}$ which can be indexed binary by three bits to indicate the presence or the absence of each of $\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3$. The resulting basis of $\bigwedge^k\mathbf{3}$ corresponds exactly to the colors and electric charges of the electron antineutrino, the *down* quark, the *up* antiquark, and the electron. Their antiparticles are represented by the exterior algebra $\bigwedge^\bullet\overline{\mathbf{3}}$ of the conjugate or dual space $\overline{\mathbf{3}}$ of $\mathbf{3}$.

The direct sum of the vector spaces $\overline{\mathbf{3}}$ and $\mathbf{3}$ is a complex 6-dimensional vector space $\overline{\mathbf{3}} \oplus \mathbf{3}$.

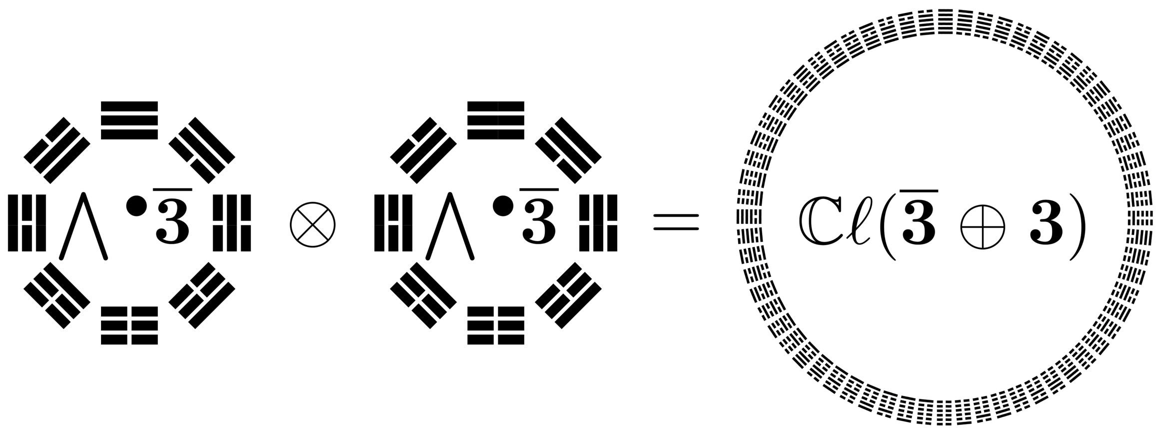



The following inner product is naturally defined on $\overline{\mathbf{3}} \oplus \mathbf{3}$, since $\overline{\mathbf{3}}$ is the dual of $\mathbf{3}$:

$$\langle u_1^\dagger + u_2, u_3^\dagger + u_4\rangle := \frac{1}{2}\left(u_1^\dagger(u_4) + u_3^\dagger(u_2)\right) \in \mathbb{C}, \tag{3}$$

where $u_1^\dagger, u_3^\dagger \in \overline{\mathbf{3}}$ and $u_2, u_4 \in \mathbf{3}$.

The inner product (3) determines a geometric algebra $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$, which is isomorphic to the algebra of complex $8 \times 8$ matrices, and has $2^6 = 64$ complex dimensions. It can also be written as $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3}) = \bigwedge^\bullet\overline{\mathbf{3}} \otimes \bigwedge^\bullet\mathbf{3}$. Therefore, its basis consists of the $2^6 = 64$ Clifford products of $\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \mathfrak{q}_1^\dagger, \mathfrak{q}_2^\dagger, \mathfrak{q}_3^\dagger$, and each element of this basis can be indexed binary by six bits.




The 8-dimensional complex vector space $\mathscr{I} := \mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})\mathfrak{q}_3^\dagger\mathfrak{q}_2^\dagger\mathfrak{q}_1^\dagger$ is a *left ideal* of $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$, that is, $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})\mathscr{I} = \mathscr{I}$. Any other left ideal $\mathscr{I}' \subseteq \mathscr{I}$ is either $\mathscr{I}$ or $\{0\}$. Then, $\mathscr{I}$ is minimal, and gives an irreducible representation of $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$. The spinors are the elements of a minimal left ideal of the Clifford algebra.

By multiplying $\mathscr{I}$ at right with elements of the basis of $\bigwedge^{\bullet}\mathbf{3}$, we get a total of eight minimal left ideals, which form a canonical decomposition of $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$ into minimal left ideals. These minimal left ideals correspond to the right colors and electric charges of leptons and quarks.

The Dirac algebra is identified with a subalgebra of $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$. Its representation on each of the eight minimal left ideals of $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$ is reducible, resulting in two representations on each ideal. In this representation, the Dirac spinors correspond to half-spinors of $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$. Consequently, each ideal contains the representation of a doublet of weakly interacting particles and the corresponding singlets, each of the two particles being half-spinors of $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$.

The symmetries of $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$ are the electromagnetic and color gauge symmetries, and are generated by operators constructed from the *ladder operators* $\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \mathfrak{q}_1^\dagger, \mathfrak{q}_2^\dagger, \mathfrak{q}_3^\dagger$[5]. Each ideal is either colorless or has a definite color or anticolor. The electric charge is not identical on each ideal, but it is identical on each half-ideal invariant to the action of the Dirac algebra. The weak, electromagnetic, and color gauge groups act by the (spinorial) adjoint representation on the algebra, and their generators are elements of $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$. As a result, the weak group acts at left on each minimal left ideal, and the color gauge group at right, permuting therefore the ideals according to the representations $\mathbf{1}_c, \mathbf{3}_c, \overline{\mathbf{1}}_c, \overline{\mathbf{3}}_c$.

In the matrix representation of $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$ shown below, the minimal left ideals correspond to columns. The Dirac algebra, the Lorentz group, and the weak symmetry act by permuting the rows according to the weak singlet and doublet representations $\mathbf{1}_w, \mathbf{2}_w$. The color symmetry acts by permuting the columns according to the representations $\mathbf{1}_c, \mathbf{3}_c, \overline{\mathbf{1}}_c, \overline{\mathbf{3}}_c$.

| | | $\mathbf{1}_c$ | $\mathbf{3}_c$ | | | $\overline{\mathbf{1}}_c$ | $\overline{\mathbf{3}}_c$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Dirac, Lorentz | $\mathbf{1}_w$ | $\nu_{R1}$ | $u^r_{R1}$ | $u^y_{R1}$ | $u^b_{R1}$ | $\bar{e}_{L1}$ | $\bar{d}^{\bar{r}}_{L1}$ | $\bar{d}^{\bar{y}}_{L1}$ | $\bar{d}^{\bar{b}}_{L1}$ |
| | | $\nu_{R2}$ | $u^r_{R2}$ | $u^y_{R2}$ | $u^b_{R2}$ | $\bar{e}_{L2}$ | $\bar{d}^{\bar{r}}_{L2}$ | $\bar{d}^{\bar{y}}_{L2}$ | $\bar{d}^{\bar{b}}_{L2}$ |
| | $\mathbf{2}_w$ | $\nu_{L1}$ | $u^r_{L1}$ | $u^y_{L1}$ | $u^b_{L1}$ | $\bar{e}_{R1}$ | $\bar{d}^{\bar{r}}_{R1}$ | $\bar{d}^{\bar{y}}_{R1}$ | $\bar{d}^{\bar{b}}_{R1}$ |
| | | $\nu_{L2}$ | $u^r_{L2}$ | $u^y_{L2}$ | $u^b_{L2}$ | $\bar{e}_{R2}$ | $\bar{d}^{\bar{r}}_{R2}$ | $\bar{d}^{\bar{y}}_{R2}$ | $\bar{d}^{\bar{b}}_{R2}$ |
| Dirac, Lorentz | | $e_{L1}$ | $d^r_{L1}$ | $d^y_{L1}$ | $d^b_{L1}$ | $\bar{\nu}_{R1}$ | $\bar{u}^{\bar{r}}_{R1}$ | $\bar{u}^{\bar{y}}_{R1}$ | $\bar{u}^{\bar{b}}_{R1}$ |
| | | $e_{L2}$ | $d^r_{L2}$ | $d^y_{L2}$ | $d^b_{L2}$ | $\bar{\nu}_{R2}$ | $\bar{u}^{\bar{r}}_{R2}$ | $\bar{u}^{\bar{y}}_{R2}$ | $\bar{u}^{\bar{b}}_{R2}$ |
| | $\mathbf{1}_w$ | $e_{R1}$ | $d^r_{R1}$ | $d^y_{R1}$ | $d^b_{R1}$ | $\bar{\nu}_{L1}$ | $\bar{u}^{\bar{r}}_{L1}$ | $\bar{u}^{\bar{y}}_{L1}$ | $\bar{u}^{\bar{b}}_{L1}$ |
| | | $e_{R2}$ | $d^r_{R2}$ | $d^y_{R2}$ | $d^b_{R2}$ | $\bar{\nu}_{L2}$ | $\bar{u}^{\bar{r}}_{L2}$ | $\bar{u}^{\bar{y}}_{L2}$ | $\bar{u}^{\bar{b}}_{L2}$ |

This model combines a generation of leptons and quarks, and their antiparticles, in the decomposition in minimal ideals of $\mathbb{C}\ell(\overline{\mathbf{3}} \oplus \mathbf{3})$, reproducing their discrete properties, but also the gauge symmetries of the Standard Model. It predicts a bare *Weinberg angle* $\theta_W$ given by $\sin^2\theta_W = 0.25$. But there are some open questions. How do we get three generations of leptons and quarks, and the continuous parameters of the Standard Model? Why this Clifford algebra of a complex 6-dimensional vector space, where does this space come from? Can this come from the geometry of spacetime, or we need some extra dimensions? If it has a geometric meaning, does this lead to a geometric unification of the Standard Model with gravity? Can it help quantizing gravity?

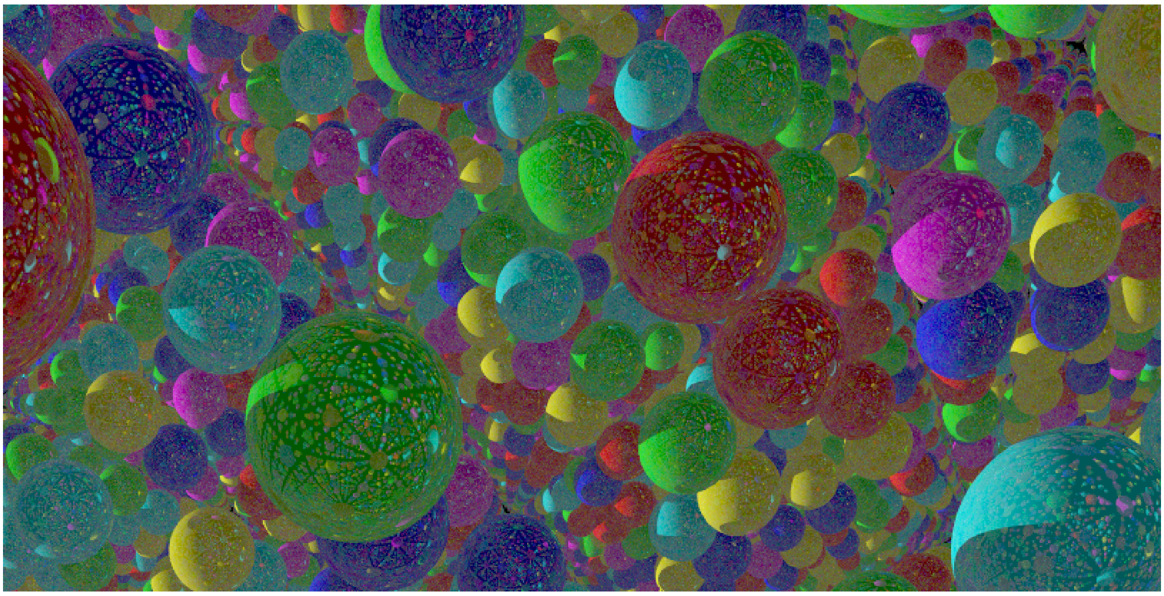
Will such a unification result in a single holomorphic field?[6]

## ☷ Indra's net

If the program whose first stages were described in the previous section will turn out to work and solve the mentioned open problems, matter will be a unified field with values in a geometric algebra, representing the leptons and quarks, and whose gauge degrees of freedom give the interactions. Maybe we will find out that this Clifford algebra has a geometric meaning and is related to the curvature in a way which gives the Einstein equation. Then, all physics will be contained in this field. Since it's dynamics is given by the Dirac operator, it may turn out to be holomorphic in a sense which generalizes the complex holomorphic functions.

If this will turn out to be the case, then the information about the whole universe is encoded at each point, in the higher order derivatives of the field at that point. So the state of the universe, including the germs at all the other points of spacetime, is encoded in the germ at each point of spacetime. Not only the field, but also spacetime itself emerges from each germ.

This resembles the metaphor of *Indra's net*, or *Indra's pearls*[7].



When we move from one point to another the germ changes, but the solution is the same. So there is a group which transforms each germ into another when moving from point to point. This group's orbits form an equivalence relation, and we can take as fundamental its equivalence classes. If there is a *multiverse*, it can be the collection of all (equivalence classes of) germs. The entire state of the universe is therefore encoded in a single class of germs.

Susan Sontag said *"Time exists in order that everything doesn't happen all at once … and space exists so that it doesn't all happen to you."* (Sontag, 2007). But in this picture, everything happens at the same point in space and time, in the germ, and is mirrored at any other point. And there is no need for a mechanism to unfold the state of the universe out of the germ, since the germ already contains everything that happens in the universe, including the observer experiencing separation in space and the flow of time. Our experience unfolds the germ, creating space and time, but the germ always remains enfolded, and we with our experiences, and spacetime itself, are always enfolded inside it. No additional mechanism is needed to unfold the germ, unfolding itself is part of the enfolded.

The idea that everything unfolds out of a germ may raise some questions about *free-will*, which I discuss in the endnotes[8].

Holomorphic fundamentalness may be a mathematically consistent basis for holism and the holographic principle, but until we will have the unified theory of physics, it remains an exercise of imagination.

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# Notes

1 The dependence of the number of particles of the acceleration of the observer is called the *Unruh effect* (Fulling, 1973; Davies, 1975; Unruh, 1976).

2 A general complex function which is analytic can be expanded in power series of both $z$ and $\overline{z}$, but the holomorphic ones don't depend on $\overline{z}$, so the power series will contain only powers of $z$.

3 The equations of physics are analytic, but this doesn't ensure that the solutions are analytic too. In the case of the Cauchy-Riemann equations, the functions satisfying it are complex holomorphic, hence analytic. The Cauchy-Riemann equations generalize, for geometric algebras, to the Dirac operator, but the equations of physics are inhomogeneous. For example, the Dirac equation contains a mass term. But the Standard Model is invariant to conformal transformations if we ignore the masses. The conformal symmetry is broken by a Higgs-like mechanism, and masses appear. So maybe this inhomogeneity is not a problem. But to be sure that after the generalization we still get something like holomorphic functions, we will need first to find the unified theory.

4 Other unified models based on geometric algebras were previously proposed by (Chisholm and Farwell, 1996) and (Trayling, 1999; Trayling and Baylis, 2001, 2004). These models are different, although they are based on Clifford algebras isomorphic as algebras (but not as Clifford algebras) with the one presented here, the geometric algebra $\mathbb{C}\ell_6$ of the complex 6-dimensional vector space. The model presented here gives the right symmetries and spin representations of fermions and quarks, and predicts a better value for the bare *Weinberg angle* $\theta_W$ given by $\sin^2 \theta_W = 0.25$.

5 The generators of the color and electromagnetic gauge symmetries and the ladder operators were previously used in a model based on the Dixon algebra $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ (Furey, 2015, 2016) and in (Günaydin and Gürsey, 1974; Barducci et al., 1977; Casalbuoni and Gatto, 1979). Also the ladder operators appear in the Spin(10) unified model (Baez, J and Huerta, J, 2010). These models are completely different, Furey's model using only a minimal left and a minimal right ideals for colors and charge, the other degrees of freedom coming from tensoring with quaternions, and resulting in a Clifford algebra $\mathbb{C}\ell_{12}$. The model presented here only needs the algebra $\mathbb{C}\ell_6$, using all of eight ideals to represent the leptons and quarks.

6 The Dirac equation differs from the Cauchy-Riemann equations by the mass term. Without the mass terms, the Standard Model is conformally invariant. Some results show that the Higgs mechanism can be obtained from conformal gravity by geometric means (Mannheim, 2012).

7 *Indra's net*, mentioned in various Buddhist texts like the *Avatamsaka Sutra*, was described in Cook (1977), chapter 1, page 2:

> "We may begin with an image which has always been the favorite Hua-yen method of exemplifying the manner in which things exist. Far away in the heavenly abode of the great god Indra, there is a wonderful net which has been hung by some cunning artificier in such a manner that stretches out infinitely in all directions. In accordance with the extravagant tastes of deities, the artificier has hung a single glittering jewel in each "eye" of the net, and since the net itself is infinite in dimension, the jewels are infinite in number. There hang the jewels, glittering like stars of the first magnitude, a wonderful sight to behold. If we now arbitrarily select one of these jewels for inspection and look closely at it, we will discover that in its polished surface there are reflected *all* the other jewels in the net, infinite in

number. Not only that, but each of the jewels reflected in this one jewel is also reflecting all the other jewels, so that there is an infinite reflecting process occurring. The Hua-yen school has been fond of this image, mentioned many times in its literature, because it symbolizes a cosmos in which there is an infinitely repeated interrelationship among all the members of the cosmos. This relationship is said to be one of simultaneous *mutual identity* and *mutual intercausality*".

8 This image of the entire universe being enfolded in a spaceless and timeless point may seem like a limitation of our freedom, even more than determinism seems to limit our freedom to those believing that the two are incompatible. Quantum mechanics seems to indicate that at least during the measurement the cold determinism of the Schrödinger equation may be broken by a discontinuous collapse in an indeterministic way, which is considered by some as allowing free-will.

But by breaking the evolution equation, the existence of discontinuous collapse would mean that physical laws otherwise always and everywhere valid are broken during quantum measurements, and also the conservation laws (Stoica, 2017a). In this case not even the laws can be fundamental. But this is not necessarily true, since it is possible that the wavefunction collapse happens by unitary evolution alone (Stoica, 2016). However, if the collapse happens by unitary evolution alone, then the past state of the quantum systems have to be very special, in order to obtain dynamically the appearance of collapse. This alternative requires thus very fine-tuned initial conditions, which seem to qualify it as *superdeterministic*. However, the block spacetime view, which is timeless, provides another way to see this: quantum measurements impose some constraints on the solution, and one should consider as physically possible only the *globally consistent* solutions (Stoica, 2015).

The idea of germs may help even more restoring the physical law, suggesting by its spaceless and timeless view that it is not as if the wavefunction converges in a predetermined way in order to collapse, but rather that the germ at the position where the collapse happens unfolds, and the wavefunction with it, into the future but also into the past.

A universe unfolding from a timeless and spaceless germ by analytic continuation imposes much stricter constraints than determinism and even than superdeterminism. Then is it possible to have free-will? It seems as if our choices are completely determined by the germ. If we want to turn the picture upside-down and consider that our choices also determine the germ, then would it be possible that our local actions determine the germ here, and by this the state of the universe everywhere? Doesn't this conflict with what other people do elsewhere, or with their will? This is not necessarily the case, because human agents can only control the germ up to some degree. We can choose what to measure, but not the outcome of the measurement, so we can't determine the complete wavefunction. The germ at a point is the collection of the derivatives of the universal function at that point, but these limitations only allow us to determine them up to some degree, and only approximately. Another agent may determine them to some degree in another place. Yet it is possible for the complete information about the derivatives to fit the choices of both agents. Or maybe each agent is free, but if their choices conflict with each other, then the germs of the two agents turn out to unfold in distinct universes, so again their choices don't conflict with each other.

For the same reasons you can't be omniscient by fully knowing the germ.

Anyway, even if there is a way to fully control a germ, you are also enfolded in the germ, and the germ is the same everywhere in a different form, so there could be no conflicting action at the fundamental level.