Pythagoras versus the Mad Tailor: an essay on the unreasonable effectiveness of mathematics

Rick Searle

The little question

Perhaps it is best from the get go to set out the positions, or at least some sufficiently narrow definition of the positions. The main issue being posed here is whether the effectiveness of mathematics in physics is trick or truth? I take the position that it is a trick as meaning that the undeniable success the use of mathematics has had for physics is somehow accidental. Mathematics, according to those holding that it is a trick is simply the most effective shorthand we have found to encapsulate the phenomena that fall under the rubric of physics. There are many reasons this type of shorthand works, but mathematics itself is not homeomorphic or isomorphic with this world, but just a type of map that is good at providing a description of some particular aspects of it.

I take the position that mathematics is the truth of physics to mean that mathematics is not a mere invention of the human mind and that its amazingly successful applications in physics should serve as proof that it is at the least a reflection of real conditions in the natural world. Mathematics is not just a way to represent the physical nature of the world that human beings have found incredibly useful, but in some sense is homeomorphic with it - a perfect map that if rendered completely could capture all of the world’s features.

The Mad Tailor

No one perhaps more colorfully captured the trick side of this debate than the science-fiction author and essayists Stanislaw Lem, with his tale of the “Mad Tailor”. For Lem, the mathematician was like a tailor who:

He makes clothes but does not know for whom. He does not think about it. Some of his clothes are spherical without any opening for legs or feet...

The tailor is only concerned with one thing: he wants them to be consistent.

He takes his clothes to a massive warehouse. If we could enter it, we would discover clothes that could fit an octopus, others fit trees, butterflies, or people.

The great majority of his clothes would not find any application. ¹

The clothes the tailor makes that actually fit someone, Lem thought, were like the mathematical tools that science used to explain nature. That is, mathematical objects could be invented that were merely fictional even if they had originally been inspired by nature but had no reference to things in the real world.

In his nonfiction work Summa Technologiae from which this tale is taken Lem was trying to tackle some really big questions such as the nature and future of science and intelligence. He was particularly interested in whether or not mathematics was the final language of science and
wanted us to explore new types of pattern languages for explaining the natural world that were
different from traditional mathematics as it had historically developed.

This is a mantle that has in part been taken up more recently by Stephen Wolfram and others
and their application of computation as a model for the phenomenon of nature and perhaps the
universe itself. It is a project of moving away from traditional mathematics as the primary
language of science that Lem conceived of even more broadly than Wolfram, asking us to look at
not merely how computers, but how biology itself processed information differently from the
way mathematics had been developed and attempt to apply these new languages to the natural
world.

**That's some robe, Pythagoras.**

The tale of the Mad Tailor has an intuitive, common sense ring to it, but our tailor gets weird
indeed if we try to retell his story in light of what has actually happened in the interplay between
mathematics and physics. It’s as if the Tailor hadn’t just made clothes that *sort of* fit his
customers, though perhaps needing a few alterations, it’s that the tailor, seemingly by chance
alone, has been able to make clothes that *exactly* match his customers measurement down to
the finest detail.

And our Tailor accidental success gets even weirder. It’s as if not only did the clothes he made
for his customers somehow magically fit precisely, it’s that he has a habit of being able to
anticipate the fashion sense of his customers decades in advance. The outfits which the
customers put together from the offerings at the Tailor's shop even sometimes have this weird
quality of knowing in advance what the customers do not, things like upcoming changes in
fashion or shifts in the weather.

The seemingly magical perfect fit is the *precision* that was the focus of Eugene Wigner’s now
famous speech on the relationship of mathematics to physics where the emphasis is not so much
on the *effectiveness*, but the incredible *precision* that mathematics gave to physics. Wigner sites
the quantum theory of Lamb shift where: “The agreement with calculation is better than one
part in a thousand.” That’s quite a tailor.

The bewildering predictive fashion sense of our Mad Tailor is the analog to the fact that
mathematics sometimes seems to build the tools science needs to tackle some problem in
physics *before* scientists have even have identified the problem these tools could be used for, as
was the case, to name just one example, with the mathematics of symmetry. And this
predictive sense is even more amazing in light of the fact that very often the equations of physics
prove smarter than we are and contain predictions that take us some time to actually prove such
as was the case most recently with the Higgs particle.

**The big and even bigger question**

The fact that mathematics has managed to be both so incredibly precise, has often anticipated
physics, and that the equations of physics so often contain predictions unforeseen by their
human authors that are only much later confirmed by observation, should certainly cause us to
seriously doubt whether the success of Lem’s Mad Tailor was accidental. The actual history of
physics itself should put to rest the argument that mathematics is a mere invention of the
human mind which we impose upon nature. Mathematics is truth not trick. Yet we’re still left
with a big question; namely, what is the relationship between this mathematics and the world it
so accurately describes?
One answer to this would be to suggest that mathematical truths are an aspect, or even the most fundamental aspect of reality, and as such are discoverable truths about the world that can be arrived at through both natural and cultural evolution.

Take the case of the humble honey bee an inspiration to mathematicians since 36 B.C. It was then that Marcus Terentius Varro proposed the Honeycomb Conjecture which postulates that the honeycomb is most efficient way to divide a surface into regions of equal size that will possess the least total perimeter. Thomas Hales was eventually able to prove this theorem correct in 1999. The honeybee seems to have discovered a mathematical truth about the world at least 23 million years before we did.

Human beings do not seem to have a naturally evolved mathematical sense that is as deep as the honeybee, there being a wide range of depth to mathematical cultures before the modern era, many of which did not go beyond mere tally systems for counting. Yet, like much else, human beings were able to discover and learn to express deep mathematical truths through cultural evolution, a story I’ll return to in a moment.

Still, even when we accept that mathematics is a deep feature of the real world we are confronted with the gap between the two that we seemingly can’t close. Even a map that is true to the territory it, or even a map written onto the territory itself, remains a mere map, and because the universe is unknowable absent our descriptions of it, the mathematical maps we use all we seem to be ultimately the only kind of truth we can ever know. Lem’s Mad Tailor, after all, was never really blind. He can see who it is he is trying to fit, while we aren’t really after just this shape, but the reality of the customer themselves.

One way that has been proposed that would overcome this problem is simply to do away with the map/territory distinction entirely in favor of the ultimate reality of the map itself. This is the case with Max Tegmark and his Mathematical Universe Hypothesis (MUH). In a way the MUH poses an even bigger big question; namely is mathematics itself the ultimate reality? How that question is answered might decide what physics and even the scientific project more broadly ultimately become.

**A (really, really) Brief History of Science and Mathematics**

To see this it is necessary to recall the history of modern physics. There’s really nothing we would recognize as physics before Newton, although even he admitted that his ideas did not emerge out of thin air, but that he “stood on the shoulders of giants”. There was a long train of development which led to the Newtonian revolution in physics that can be traced all the way back to the Sumerian invention of the partial script that would become our mathematics between 3,500- 3,000 B.C.E. an invention that released mathematical thinking from the limited memory capacity of the human brain.

After the Sumerians, you get the ancient Greeks who not only gave us the first attempts at purely materialistic explanations for natural phenomenon, but the concept of proof in mathematics, the systematic study of geometry, and even the Pythagorean idea that mathematics was the language of nature. Pythagoras, of course, inspired Plato who gave us the idea that the kinds of mathematical truths the Greeks were uncovering were both discovered and timeless. Plato also gave us the idea that these “forms” were the fundamental aspect of existence, and something more real than the world we experienced through our senses.
The Dark Ages that followed the fall of Rome were perhaps not as dark as some have claimed, but clearly part of the motivation for many of the early figures that began the scientific revolution in the 16th century was to move away from theological and metaphysical explanations for what occurred in the natural world back to the types of materialistic theories pioneered by the ancient Greeks that added the new elements of both being based on observation and the couching of those explanations in the language of the new mathematics.  

Mathematics itself had advanced slowly since the breakthroughs of the Greeks and had taken over a millennia to arrive at non-cumbersome number systems that allowed things we would consider elementary, like say ease of multiplication and division, not to mention, non-symbolic algebra, symbolic algebra, and algebraic-geometry.

All of this is a story that is very well known. What is less well known is that when Newton built off of these developments, invented his version of the calculus, and with the publication of his *Principia Mathematica* gave birth to modern physics, many of his fellow scientific revolutionaries, not without justice, labeled him an occultists who was undermining their project to find new, purely materialistic theories to describe the natural world.

The reason for this accusation is that Newton proposed no *mechanism* by which the gravitational force he postulated worked. To many of his contemporaries it looked like just numbers an elegant mathematical description to be sure, but one that lacked any grounding in something *real*. His equations provided no hypothesis regarding *why* they worked.

Newton was not all that concerned regarding such accusations. He had observed phenomenon and created a mathematical model that captured them. His theories matched future observation with unparalleled precision. Perhaps there was a materialistic explanation that explained his theory, perhaps not.

Physics has never really left this state of being able to use mathematical descriptions of reality that are enormously successful at making predictions, which is also coupled with the fact that these descriptions at rock bottom are empty of any underlying referents that we can grab hold of other than the equations themselves. Einstein’s space-time is a purely mathematical object, as are the phenomenon captured in the equations of quantum mechanics.

Yet with the exception perhaps of chemistry, progress in the other sciences has largely proceeded from continuing the scientific project as understood by Newton’s materialist opponents, of trying to more clearly define the *what* we are dealing with and where purely mathematical descriptions that do not add such clarification are not on par with theories explaining why exactly things work in such and such a way and not some other. The appearance of the MUH might be important because it signals that this historical divergence between physics and the rest of the sciences is finally approaching a conclusion.

**Weak MUH and the future of physics**

There is more than one reason to think that the emergence of the MUH signals that the division over the fundamental nature of mathematics between physics and the other sciences is now groping its way towards a conclusion among which I would list everything from the emergence of the study of mathematical reasoning by neuroscience, to the ubiquity of high powered computers that allow sophisticated mathematical modeling for sciences far removed from physics. Yet at rock bottom these and above all the MUH itself signals that the ancient philosophical dispute between those who hold that mathematics is real and those who hold it to
be a convention— the debate between Pythagoras and Lem’s Mad Tailor, or Platonists and anti-Platonists has become a scientific question. And much the same as was the case in for other long lived philosophical debates over the origins of life and the beginning of the universe, science will ultimately resolve such questions.

There is, however, a problem that I think arises if we try to use the MUH exactly as Tegmark has given it to us in order to uncover how the debate between those who hold mathematical objects to be real and those who believe them to be human conventions might be scientifically resolved. The problem stems from the fact that the MUH is formulated in the context multiverse theories. For if there is a very large, perhaps infinite number of possible mathematical structures, how can we be sure we have stumbled upon the right one? In being able to answer definitively the core philosophical dispute over the reality of mathematics we return to our bafflement over mathematics “unreasonable effectiveness” for physics, or how, among such a huge number of mathematical structures are we able to find the one that is actually ours?

To those who might argue that any version of the MUH requires some version of the multiverse theory because otherwise we’ll be faced with the old accusation against Platonism as to where mathematical truths that have not been discovered by human beings reside I will pose what follows. That even if there is only one universe all mathematical truths, including undiscovered ones, can be said to be embedded in at least one of three places in that universe: the natural world (as was the case for the honeybee’s solution to the packing problem before we discovered it), the human mind along with other types of minds in the cosmos, should they exist, or in the range of possibilities that emerge one once defines some set of constraints (say defines a triangle as a two-dimensional shape with three sides) which might characterize a large portion of the actual entities studied by pure mathematics along with games such as chess.

I now return to proposing a weaker version of the MUH which might allow us to both resolve the divergence question between physics and the rest of science that might provide us with an eventual route to answer both the question of mathematics unreasonable effectiveness for physics and the question of whether or not the Platonic form of mathematical truth represent the sole and ultimate reality. This weaker MUH would look like this:

*We live in a mathematical structure that is fully homeomorphic with a language of mathematics that retains this mathematics’ Platonic features i.e. it is timeless and reversibility.*

Developments within mathematics, physics and even broader science itself that should result in confidence or doubt regarding a weak MUH would likely include one or more of the following:

*Our confidence in a weak MUH should diminish the longer more than one mathematical model of physics are considered equally effective to explain the same phenomenon.*

This is because having more than one “map” that describes reality that prove non-combinable should lead us to question whether such maps can ever be considered synonymous with the phenomenon they describe.

*Our confidence in a weak MUH should increase the more physics appears to converge on one mathematical model constructed on the basis of traditional mathematics with Platonic i.e timeless and reversible mathematics.*
This is because the closer we come to a single equation that fully captures the universe the more confident we can be that there is no fundamental gap between the mathematical structure it describes and the universe we inhabit.

Our confidence in a weak MUH should dramatically decrease should equally or more effective models for physics emerge which are constructed on the basis of alternative forms of mathematics that do not share traditional mathematics Platonic i.e. timeless and reversible features.

None of these alternative models, such as Stephen Wolfram’s version of a physics built on the discrete and irreversible mathematics of computation, seem likely to challenge the success of traditional mathematics based on equations as the basis of physics anytime soon, but the possibility is out there. Should it be the case that such non-traditional models prove fruitful and competitive with traditional mathematics, this would surely be bad news for even a weak version of the MUH, for it would suggest that our current success using traditional mathematics has indeed been a mere historical accident. Lem and his Mad Tailor will have struck back.

Our confidence in a weak MUH should decrease should there be notable theoretical and practical progress in other scientific fields that have embraced alternative or ad hoc mathematical models compared to those of a physics committed to Platonic i.e timeless and reversible mathematics and the search for an overarching theory becoming stuck in a state of theoretical and practical stagnation.

This would be perhaps the worst case scenario for not just physics but for the whole scientific project to make the world intelligible. We might gain extraordinary technological prowess through the construction of mathematical models that capture phenomenon, some of which might be constructs of computers alone, and yet have no idea why those models work or how we could possibly mesh them together in some overarching theory. We see some hints of this in the way artificial intelligence is preceding, and AI itself may prove the primary source of mathematical models that work, but whose underlying reasons we find unintelligible.

The future development of physics, and to some extent the other sciences, will ultimately decide if Newton’s mathematical intuition and an at least weak version of the MUH are correct. We will also likely learn this century whether it has always been possible to build models of physics on par with our current ones out of very different types of mathematics with entirely different intellectual genealogies, which either confirm decide intuitions about the nature of mathematics and the world stretching back to Pythagoras, or show that the Mad Tailor was not so mad after all.
References


13 Ibid.

