The fundamental distinction between Mathematics and Physics, arises from the distinctly different nature of their “starting points”. The usefulness of mathematical methods, for describing observable behaviors, depends upon the complexity of those behaviors, which in turn depends upon their information content.

I first encountered the principle of Reductio ad Absurdum, when I was in High School. For a long time, it puzzled me. The generic form of the principle is something like: “If A is true, then B must also be true, but, the conclusion that B is true, is self-evidently absurd, therefore, A must not be true.” But why blame the failure of an argument, only on its starting point, or initial assumption? Perhaps the absurdity arose from a faulty application of the rules for deductive logic. Eventually, it occurred to me, that the very definition of a “well reasoned” argument, is that there are no such faulty applications of the logic, within the argument. Hence, the only remaining source for the absurdity, in any “well reasoned” argument, is the argument’s starting point. This brings us to the fundamental distinction between Mathematics and Physics; the nature of their starting points.

It is not a question of “Trick or Truth”, as the theme of this essay contest presupposes. Rather, it is a question of either “Interesting or True” starting points. No one cares if the starting axioms and postulates of Mathematics are “True”. Their value, or goodness, is not measured by their “truth”. They are deemed to be “good”, and “interesting”, if they lead to interesting conclusions. There is no other truth to the matter, for such starting points.

Physics is entirely different. Physics is supposed to describe how the world behaves. It either does so, in “Truth”, or else it fails. Consequently, unlike Mathematics, people do care about the “Truth” of the starting points. If bad initial assumptions, followed by “well reasoned” arguments, leads to a faulty description of the world, then, unlike the case in Mathematics, one can declare, that, while the starting point may or may not be “Interesting”, it cannot be “True”. For example, if one assumes that the probability of a sequence of events occurring, should be computed via the rules for independent events, the resulting probability will not yield a “True” depiction of the world, if the events are not, in “Truth”, independent, even when all the rules are applied correctly. The truth is, that the truth matters, in Physics. But not in Mathematics. At least not in so far as only the starting points are being considered.

This truth does indeed matter. Four hundred years ago, Francis Bacon, in “The New Organon”, noted that Natural Philosophy (Science) had stagnated for 2000 years, partially as a result of the ancient greeks modeling science on mathematics (Deductive logic, rather than Inductive logic), and consequently not being sufficiently motivated to verify the truth of their starting points, via observations. After all, such truths do not matter in Mathematics. Present day observers, have begun to make similar observations, regarding stagnation in contemporary theoretical Physics, and for the same reason; Too much emphasis on the beauty of the math, and too little emphasis on the demonstrable truth of the starting points.

So where should one start, in the search for anything that might be either “interesting” or “true”, regarding the connections between Mathematics and Physics? One obvious starting
point, is with the observation that mathematics seems “unreasonably” effective in describing the observations of concern to physicists, but much less so, in most other fields. What are the characteristics of these fields, that correlate with this fact? In a word, complexity. Or, more precisely, the effectiveness strongly correlates with the information content of the observational data, that the mathematics is being used to quantitatively describe; the lower the information content, the more effective mathematics is, at precisely describing it. Physics deals only with very low information content phenomenon (the “laws” of physics do not require very many bits, or other symbols, to specify them) in comparison with phenomenon at the opposite end of the complexity spectrum, like life. Consequently, it is not surprising that a low information content symbology, like sets of mathematical equations, succeed at capturing the essence (the information content) of one, but not the other.

But exactly what is information? Those with a propensity for using mathematics, to describe things, usually attempt to define information using mathematics. Unfortunately, information can be complex, which results in cases in which its essence cannot be described via short sequences of symbols. The quantity of information present might be so described, but not the information itself.

Claude Shannon, the founder of Information Theory, noted that information is not even an objective “thing”. It is subjective. From Shannon's perspective, the amount of information that needs to be conveyed, within a message, to enable a receiver of the message, to reconstruct the complete message, without errors, depends on how much relevant information the intended receiver already knows; one never needs to convey information that is already within the receiver's possession. Since different receivers might possess differing amounts of relevant information, the amount of information required to be conveyed, may differ substantially, from one receiver, to another. What is information for one, is useless redundancy, to another. Hence, information is subjective.

The most striking example of this, can be found in discussions about a highly complex phenomenon; human-like intelligence. John Searle's famous argument about the “Chinese Room”, boils down to the fact that an entity in possession of ALL relevant information, regarding ALL possible messages, can successfully decode ALL messages, as if all messages consist of nothing more than a single, long index-number, which simply serves to specify which response, stored within the receiver's vast memory, should be recalled from memory. Such a system exhibits a complete disconnect between physics and behavior, because the system treats ALL inputs, not as measurements, but as indices. Measurements have most and least significant digits. Indices do not. If there is one bit wrong, anywhere within even a huge message, then it codes for a different index, and thus a different message, and a catastrophic failure will occur in the message reconstruction/look-up. A less extreme form of this type of failure, is familiar to anyone who has compared the noisy “snow” slowly degrading the image quality of an analog TV picture, as the noise increases, with the failure modes of an HDTV receiving over-the-air broadcasts. The analog TV treats the received signal as measurements of intensities, used to “paint” the screen. But the HDTV treats them as symbol sequences, in which any error in a received sequence, may result in dramatic image reconstruction failures.

Such systems respond to all inputs symbolically, rather than physically. And their detailed behavior cannot be described with any short set of equations. This is why human observers behave in ways that cannot be successfully modeled, by mathematical, quantum
measurement theories, that work perfectly, for less complex observables. Hence the paradoxes of why observers and observables seem to be rather different beasts, even though they both ultimately obey the same laws of physics. The physics is rendered superfluous, when complexity enables entities to respond symbolically, rather than physically, to many of the observables within their environment. Cars do not stop when encountering red traffic signals, for the same reason they stop when they encounter a concrete barrier. One is a symbolic response, the other physical. Knowing all the physics about electromagnetic waves, and automobiles, is not going to enable one to predict that it is the red light, rather than some other color, that will cause the car to stop.

Mathematics has its limits. Even Godel's famous Incompleteness Theory, about the limitations of math, is related to the information content of math's starting-points. If there are only a finite number of axioms being used as starting points, each expressed with a finite number of symbols, then they contain only a finite amount of information, enabling the construction of only a finite length index number, which cannot be used to look-up the answers to an infinite number of unique questions, concerning the validity of mathematical theorems. Information content also informs questions about the nature of randomness. Is a sequence of numbers "Truly" random, or merely pseudo-random? Can one determine if a sequence is pseudo-random, if the algorithm is of cryptographic strength? Searle's Chinese Room can, since it knows everything. And so can any other observer, with much less information than a Chinese Room, as long as the observer has all the relevant information. The creators of a cryptographic, pseudo-random number generator, can certainly recognize the messages produced by their own creation, even though it was created precisely to prevent others, from doing the same. As Shannon observed, information is subjective, what is required by one observer, in order to answer any given question, may be irrelevant to another. What is uncertain for one observer, with limited, relevant a priori information, may be easily deduced by another, with more information. The Fourier Uncertainty Principle, does not limit the accuracy with which an FM receiver can measure the frequency of a frequency modulated signal, because the receiver knows that a Fourier Transform based Measurement Model, can be profitably replaced, with an entirely different measurement model, based on the concept of "Instantaneous Frequency", rather than Fourier superpositions. It can do this, precisely because it knows, a priori, that the signal was designed specifically to enable it to be done. In other words, the message/signal does not have to convey enough information for an observer to deduce how to decode/decrypt it, if the observer already knows how to do that. A less fortunate observer cannot exploit this simple fact.

In short, a harmony exists between mathematics and physics, whenever the information carrying capacity of the symbols being used by math, to describe physical measurements, matches the information content of the measurements being described. Hence, one does not need complex math, to describe a single, constant measurement. And no humanly comprehensible amount of mathematical equations, will enable one to mathematical describe, in detail, a truly complex physical process, like life and consciousness.

Robert H. McEachern was educated as an AstroPhysicist. He then worked for several years as a Geophysicist, during which time he became interested in signal processing theory. He then spent the rest of his career developing signal processing algorithms for application to communications systems, and sensor systems. He is now retired.