A Systems Theoretic Approach to Physics

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It can be argued that the Holy Grail of physics is to develop a model of physical processes based on first principles. That is, a simple set of rules, or axioms, is presented from which a complete description of physics is unavoidably emergent. It is in the spirit of this quest that an axiomatic system is presented in this paper. The system that arises from two very simple axioms is an example of a general system. The paper makes no specific claim that this system describes the empirical nature of physics, but rather lays out a set of theorems and constants that applies to all systems that satisfy the two defining axioms, of which physics may be one. The system defined by these axioms is then analyzed and is shown to have a remarkably robust set of properties and behaviors. Even more remarkably the theorems, relations, and constants that arise out of the axiomatic system appear to map one-to-one to the set of equations and constants that are commonly used to describe physical processes. This paper will derive from first principles three specific equations/relations that map to physics, analogs of the Lorentz transform, the uncertainty principle, and the Einstein field equations. It is argued that the common threads that exist between the theorems of the abstract general system and the equations, relations, and constants that define most of modern physics provide a strong foundation upon which to explore the deeper relationship between mathematics and physics. As the general system is both discrete and non-deterministic, it may then be argued that physics shares these same characteristics. Some differences between the formalisms of the general system and traditional quantum mechanics and general relativity are noted. These differences may provide an empirical trail on which the applicability of the general system to physics may be evaluated.

1 The Axiomatic System

A formal system $S$ is defined that satisfies the following two axioms:

- The general system $S$ is described as a random walk in a single dimension
- Associated with the system is a consistent metric: let the distance to the $n^{th}$ state transition in the system $S$ be proportional to $n$

It will be shown that the following equation is a theorem in the system $S$:

$$t = \tau \sqrt{1 - \frac{v^2}{c^2}}$$

The set of symbols $t, \tau, c,$ and $v$ are arbitrary labels chosen to emphasize a possible relationship between the general system and physics. Although the properties and behaviors of each of these symbols are not explored in this paper, the derivation of their genesis will be. Further reflection may indicate that a semantic interpretation of each of these symbols maps very closely, if not exactly, to the meanings of these systems as used in traditional physics. In some cases the implied semantics of these symbols appear to map more closely to empirical behaviors than do the standard interpretations. For example, the concept of time that is presented in this paper is monotonically increasing and is not reversible. On the surface, this behavior appears to more accurately characterize time than do more traditional treatments.

It important to keep in mind that the equation listed above does not represent a Lorentz transformation. The notion of a reference frame is undefined. The $t$ in the equation may have
nothing whatsoever to do with time (although it could). The $\tau$ in the equation may have nothing whatsoever to do with proper time. However, the relationship between $\tau$ and $t$ has everything to do with the mathematical relationships that underlie both physics and the general system presented in this paper.

It should be noted that the behavior of the general system $S$ provides a valid model for other systems besides physics. Indeed, within every system that can be described by the two axioms there must exist something analogous to a Lorentz transformation.

As the formalisms of the system $S$ are explored a complex geometry arises out of the analysis. When the metric axiom, the second of the two axioms, is applied to the system $S$ with vigor it will be shown that two additional degrees of freedom must be extended from the original one-dimensional manifold in order to allow for a system that is consistent with the two axioms.

The resulting Riemannian 3-manifold (not a pseudo-Riemannian manifold) is then shown to have a non-zero curvature that is described by the equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \alpha T_{\mu\nu}$$

Where $\alpha$ is a scalar and $T_{\mu\nu}$ is not the traditional energy-momentum-stress tensor, but rather a more general concept of which the energy-momentum-stress tensor may be but a good example. Again, any and every system based upon the two axioms must incorporate a variant of this equation as part of the description of the overall system.

Finally, it will shown that when the curvature of the system $S$ is small the logic of the metric axiom can be best approximated by the relation:

$$\Delta x \Delta p \geq \hbar/2$$

As with the other two theorems in $S$ the meaning of the symbols $x, p,$ and $\hbar$ are not synonymous with the usual notions of position, momentum or reduced Planck’s constant. They are all generalizations of these characteristics and will be shown to all be manifestations of the proportionality constant.

Although it is outside the scope of this paper, it can be shown that relation $\Delta x \Delta p \geq \hbar/2$ can be generalized to be consistent with a highly curved geometry. It is hoped that this generalization can serve as the basis for a mathematically consistent model of quantum gravity.

These three particular theorems of the general system $S$ were chosen to emphasize that equations/relations that look remarkably similar to the Lorentz transformation, the Einstein field equations, and the uncertainty principle are unavoidably emergent from first principles in the system $S$. That all three of these theorems are emergent from the axioms suggests that all three share a common underlying mathematical framework. Physics may be a manifestation of this general system.

It must be understood that the system $S$ is a formal system. As such, the burdens of formal proof are high. Because of the limited scope of this paper rigorous proofs will not be presented. However, a general description of the logic of the proofs will be outlined. It should be noted that a more thorough and detailed analysis of the system $S$ is available.

Although the symmetries embodied in the system suggest a stable configuration of three spatial and one temporal dimension, in order to simplify the mathematics the formalisms used in this paper consider systems with only a single spatial and a single temporal dimension.
2 Let the Magic Begin

The system $S$ is a random walk. Let a system of $n$ state transitions, called events, be labeled $S(n)$. The notation $\beta(n)$ represents the state of the system $S$ after $n$ events $S(n)$. The metric axiom states that the distance to the $n^{th}$ event in the system must be proportional to $n$. Label the proportionality constant $\overline{c}$.

Examine a system $S$ representing a single event. This system is labeled $S(1)$. The label $\beta(1)$ represents the first, and in this case final state of the system $S(1)$. Since the system $S$ is a Markov process, an initial state of 0 implies the final state of the system $S(1)$, is either $\beta(1) = 1$ or $\beta(1) = -1$. The state of the system $S(1)$ can be represented by the set of coordinates $S(1) = (1,0, -1)$. The system $S(1)$ is clearly a random walk. The metric axiom demands that the distance between the coordinates 1 and 0 and 0 and -1 both be equal to $\overline{c} = 1 * \overline{c}$. The system $S(1)$ satisfies the metric axiom and therefore is a valid example of the system $S$.

Now examine the system $S(2)$. The notation $S(2)$ describes an instance of the system $S$ of 2 events. The state of system $S(2)$ must be in one of the states: $S(2) = \{2,1,0,-1,-2\}$. The probability that the system is in state 1 or -1 is zero. This assertion is obvious and a proof will not be presented.

Therefore, the final state of the system $S(2)$, $\beta(2)$, must be a member of the set: $\{2,0,-2\}$. If $\beta(2) = -2$ or $\beta(2) = 2$ then the second axiom is satisfied as the distance between the coordinates 0 and 2 and 0 and -2 is $2\overline{c}$. The distance to the second event is proportional to the number of events, in this case 2.

However, if the final state of the system $S(2)$ is 0, $\beta(2) = 0$, then the metric axiom is not satisfied. The distance between the initial state and the final state is 0 and not $2\overline{c}$.

Because the system $S$ is axiomatic it is necessary to attempt to extend the geometry and associated coordinate system in order to accommodate the axioms. If it is not possible to extend the scope of the system, then the axioms define a system that is inconsistent. Fortunately, in this instance it is indeed possible to expand the geometry in order to meet the axiomatic rules and maintain system consistency.

The only way to satisfy the axioms is to expand the coordinate system to include a second degree of freedom. As such, assign the state of $S(2)$, where $\beta(2) = 0$, to the complex coordinate $(0,2)$, where the X-coordinate represents the state of the system $\beta(2)$ and the Y-coordinate represents an imaginary state of the system, $i\beta(2)$. The complete state of the system must be represented by a complex number $(\beta(2) = 0, i\beta(2) = 2)$, rather than a real number. Furthermore, the metric demands that the distance from the coordinate $(0,0)$ to the coordinate $(0,2)$ must be equal to $2\overline{c}$.

The set of all possible states of $S(2)$ lie on a semi-circle with a radius of $2\overline{c}$ and origin $(0,0)$.

Let the geometry associated with the system $S$ be described by the differential manifold $\mathcal{M}_S$.

It is a theorem in $S$ that the system $S(n)$ contains exactly $n + 1$ allowable coordinates, or allowable states. Allowable states are those states that are associated with non-zero probabilities. It is also provable that the union of all $S(n)$ is the northern half of the complex plane and that intersection of any two instances $S(n)$ and $S(m)$, where $n \neq m$, is the null set. The system $S$ is an example of a complex vector space and also has the property of being affine. These proofs are relatively simple, but will not be presented in this paper.

2.1.1 Analysis of Displacements in the X Dimension

It is a theorem in $S$ that all displacements along the X-axis are proportional to the number of events.
All coordinates that lie on the X-axis are of the form \((\beta(n), 0)\). That is, the displacement in the Y dimension is 0. Therefore, in order to satisfy the metric axiom, the displacement on the X-axis to the coordinate \((\beta(n), 0)\) must be equal to:

\[
x = \beta(n) \ast c
\]  

(1)

### 2.1.2 Analysis of Displacements in the Y Dimension

Unlike displacements in the X-dimension, in general, displacements in the Y dimension are not proportional to \(n\). The displacements in the Y dimension are such as to ensure that metric axiom, *let the distance to the \(n^{th}\) state transition in \(S\) be proportional to \(n\)*, holds true.

Because the system \(S\) is axiomatically defined and there is no geometrical obstruction, the intrinsic curvature of the manifold is a priori defined to be 0 and therefore the Pythagorean theorem is valid in the system \(S\). And so, letting \(x\) and \(y\) be displacements, \(n \ast c = \sqrt{x^2 + y^2}\). Solving for \(y\):

\[
y = \sqrt{(n c)^2 - (x)^2}
\]

But it was previously determined that the displacement in the X dimension is: \(x = \beta(n) \ast c\).

And so,

\[
y = \sqrt{(n c)^2 - (\beta(n) \ast c)^2}
\]

Or,

\[
y = n c \sqrt{1 - (\beta(n)/n)^2}
\]

Given that a system of \(n\) events is in the state \(\beta(n)\).

Let the ratio \(\beta(n)/n\) be called the *displacement ratio*, or \(R_P\). Then,

\[
y = n c \sqrt{1 - (R_P)^2}
\]

(2)

### 2.2 Analysis of the Angular Displacement

Each integer valued allowable state \((\beta(n), i\beta(n))\) for the system \(S(n)\) lies on a semi-circle with a radius of \(n c\). As such, each integer valued coordinate \((\beta(n), i\beta(n))\) is associated with a unique angular displacement that is function of \(R_P\). Call this angular displacement \(\omega\), where \(\omega = \arccos(R_P)\).

### 3 Analysis of the Polar Coordinate Representation of the System \(S\)

Every coordinate \((\beta(n), i\beta(n))\) can also be described in polar notation as the product of a polar angle \(\theta\) and a radius \(r\), \((\beta(n), i\beta(n)) = \theta \ast r\), where \(r\) is an integer multiple of \(c\). Stating that the radius \(r\) may only take on discrete values such that \(r = n c\) is equivalent to describing a circle with a circumference of \(C = 2 * (n c) * \pi\), where \(n\) is an integer. Since the system \(S\) is only defined for the northern complex half-plane, this is equivalent to stating that all allowable coordinates must lie on a semi-circle with an arc length \(s = n c \ast \pi\). The equation for the arc length of all allowable semi-circles may be re-written as \(s = n \ast (c \pi)\).

Page 4 of 9
For example, the arc length of the system $S(1)$ must be $s = 1 \times (\pi\tilde{c})$; the arc length of the system $S(2)$ must be $s = 2 \times (\pi\tilde{c})$; and so on. Further generalizing, the arc length of the system $S(n)$ must be $s = n \times (\pi\tilde{c})$.

As previously shown, there are exactly $n + 1$ allowable coordinates with non-zero probabilities associated with the system $S(n)$. When translated from Cartesian to polar coordinates the metric axiom demands that the distance between adjacent allowable coordinates for all systems $S(n)$ must be $s = \tilde{c} \times \pi$. In order to maintain the constant arc length $\tilde{c} \times \pi$, as the radius $r$ grows the polar angle $\Delta \theta$ must shrink proportionally such that $r \Delta \theta = \tilde{c} \times \pi$, where $\tilde{c} \times \pi$ is the arc length subtended by the angle $\Delta \theta$. The arc length of the subtended angle $\Delta \theta$ may not be smaller than $\tilde{c} \times \pi$, but it could certainly larger, specifically integer multiples of $n$ larger. The equation can be generalized to the relation $r \Delta \theta \geq \tilde{c} \times \pi$.

Because the manifold $\mathcal{M}_S$ is an affine space meaning there is no preferred origin the equation may be further generalized to state that the distance between any two allowable states $\Delta \omega$ in the system $S(n)$ must satisfy the relation:

$$\Delta r \Delta \theta \geq \tilde{c} \times \pi$$

But it will now be shown that arc length of adjacent allowable states $\Delta \omega$ is almost always less than $\tilde{c} \times \pi$. This inconsistency threatens the validity of the axiomatic system.

### 3.1 Comparison of $\omega$ and $\theta$

It has been shown that there are $n + 1$ polar angles $\omega$ and $n + 1$ polar angles $\theta$ for the system $S(n)$. There is a one-to-one correspondence between the set of angles $\omega$ and the set of angles $\theta$. The set of angles $\omega$ points to the set of allowable coordinates for $S(n)$. The set of angles $\theta$ defines the set of $n$ arc-lengths of $\tilde{c} \times \pi$ that sit on the allowable semi-circle associated with $S(n)$. However, an analysis shows that $\theta \neq \omega$ for all angles except along the Y-axis ($R_D = 0$) and that the two sets of angles differ by a factor of $1 / \sqrt{1 - (R_D)^2}$. More importantly, but for 2 exceptions near the X-axis, the arc length associated with every pair of adjacent allowable states $\Delta \omega$ in the system $S(n)$, where $n > 2$, is less than $\tilde{c} \times \pi$, $\Delta r \Delta \omega < \tilde{c} \times \pi$. This is a direct violation of the metric axiom, as described above.

Once again an apparent inconsistency has appeared in the analysis. And once again, this apparent inconsistency can be corrected by extending the manifold into another degree of freedom.

### 3.2 The Extension of the Manifold $\mathcal{M}_S$ into the Z-dimension

There is no requirement that the set of angles $\omega$ be identical to the set of angles $\theta$. Instead, the metric requires that there be a set of angles $\omega$ and a set of arc lengths connecting adjacent angles $\Delta \omega$ such that each arc length has a distance of $\tilde{c} \times \pi$. These two constraints can be accommodated if and only if the underlying differential manifold $\mathcal{M}_S$ is curved. In order to account for the term, $1 / \sqrt{1 - (R_D)^2}$, called the displacement factor $\gamma$, it is necessary to add another degree of freedom to the system $S$.

As described before, when the metric axiom required the addition of an extra degree of freedom, the system $S$ was forced to expand the manifold into a 2nd dimension, the imaginary state $i\beta(n)$. In the same manner the only way to reconcile this violation of the metric is to again expand the manifold $\mathcal{M}_S$, this time into a 3rd dimension, the Z-dimension, to account for the displacement factor.

The displacement in the Z dimension is minimum along the Y-axis. As the angle $\omega$ grows farther from the Y-axis the value of the displacement in the Z-dimension increases, as does the discrepancy between $\omega$ and $\theta$. 
If the displacement in the Z-dimension along the Y-axis \((\omega = \pi/2, \text{or } R_D = 0)\) is \(z_0\) then the general displacement in the Z-dimension for all values of \(\omega\) must be \(z = z_0/\sqrt{1 - (R_D)^2}\).

Also, it is important to notice that when \(R_D\) is small then \(\omega \approx \theta\) and so \(\Delta r \Delta \theta \approx \Delta r \Delta \omega\). The relation \(\Delta r \Delta \omega \geq c^* \pi\) can then be transformed into Cartesian coordinates as \(\Delta x \Delta R_D \geq c/2\).

### 3.3 nis of the Basic Metric

As originally defined, the metric describes the distance between the origin \((0, 0, 0)\) and an allowable state \((\beta (n), i\beta (n))\). However, since the extension of the geometry into a 3rd dimension the metric axiom must be reinterpreted. The metric axiom now demands that the distance between the origin \((0, 0, 0)\) and the allowable system state \((\beta (n), i\beta (n), z)\) must be an integer multiple of \(c\).

### 3.4 A Description of the Curvature

A visual description of the manifold now looks very much like a valley that starts up with extremely steep sides near the origin but then flattens out in the distance with \(z\) having a minimum value, \(z_0\), along the valley floor the (the Y-axis) and maximum values close to the X-axis. The curvature of the manifold is small for large values of \(y\) near the Y-axis. The curvature is large near the X-axis and the origin. Rectangles of length \(nc\) and height \(z\) radiate out from the origin in the manner of a spiral staircase. The value of \(z\) is invariant along a constant angle of \(\omega\). Label these rectangles momentum, or \(p\). Even though these objects are rectangular a detailed analysis beyond the scope of this paper oddly shows that momentum is best described as a tensor \(p_{\omega p}\). The covariant component \(\omega\) represents the angle associated with the displacement ratio \(R_D\), while the covariant component \(\varphi\) represents the angle defined by \(\arcsin\left(\frac{z}{nc}\right)\).

For very small values of \(z/nc\) the tensor \(p_{\omega p}\) behaves as a vector. For values of \(z/nc > 1\) the tensor is degenerate.

Letting momentum be represented as a vector \(p\) allows the geometry of the manifold to be described by the rank-2 tensor that results from the tensor product of any two of the rectangles that radiate out at different angles from the origin. Label one of the momentum rectangles \(p_\mu\) and the other momentum rectangle \(p_\nu\). Call the resulting tensor \(\tau_{\mu\nu}\), where \(p_\mu \otimes p_\nu = \tau_{\mu\nu}\).

The curvature of the manifold \(M_G\) can be described by the Riemann curvature tensor \(R_{\sigma\mu\nu}^\rho\). The various symmetries that characterize \(\tau_{\mu\nu}\) lead to the following theorem.

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \alpha \tau_{\mu\nu}
\]  

(4)

Where \(\alpha\) is a scalar, \(R_{\mu\nu}\) is a variant of the Riemann curvature tensor called the Ricci tensor, \(R\) is the Riemann curvature scalar, and \(g_{\mu\nu}\) is the metric tensor. While the proof of this theorem is non-trivial, it is well known by most physicists.

### 4 Dimensional Analysis of the Coordinate System

To this point the displacements and distances defined within the context of system \(S\) are written in terms of the dimensionless constant \(c\). The system does not recognize any units of measurement associated with any of the three axes. Yet, in order to map the abstract general system into a specific physical, or other, system it is necessary to associate units of measurement with each of the axes.
Arbitrary units of measurement will now be associated with the X, Y, and Z-axes as an initial step towards mapping the abstract general system $S$ to the discipline of physics. These units of measurement carry no semantic content. They are merely symbols. Although devoid of semantic content, the symbols used for describing measurement are carefully chosen to emphasize their applicability to physics.

Let the notion of an abstract measurement unit be indicated by enclosing the unit of measure in brackets. Thus $k_m(m)$ represents $k_m$ units of the abstract unit of measurement ($m$).

### 4.1 The Assignation of Dimensionality

Let the displacement in the X dimension be measured in units called $m$. Let the dimensionality of a displacement in the X dimension be called Length or $L$. Let $\vec{c} = k_m(m)$ describe a displacement of $\vec{c}$ in the X dimension in units of $m$. Let a unit distance of $k_m$, 1 * $\vec{c}$, be called a Planck length, $\ell_p$, where: $\ell_p = k_m(m) = 1.16162 \times 10^{-35} \, (m)$. The value of the numeric assignation is completely arbitrary. Let the value of the maximum displacement on the <distance axis> for a system of $n$ events $S(n)$, $n * \vec{c}$, be called a proper length, $L_0$, or: $L_0 = nk_m(m) = n \ell_p$.

Let the displacement in the Y dimension be measured in units called $s$. Let the dimensionality of a displacement in the Y dimension be called Time or $T$. Let $\vec{c} = k_s(s)$ describe a displacement of $\vec{c}$ in the Y dimension in units of $s$. Let a unit distance of $k_s$, 1 * $\vec{c}$, be called a Planck time, $t_p$, where: $t_p = k_s(s) = 5.39124 \times 10^{-44} \, (s)$. This assignation is completely arbitrary. Let the value of the maximum displacement on the <time axis> for a system of $n$ events $S(n)$, $n * \vec{c}$, be called a proper time, $\tau$, or: $\tau = nk_s(s) = nt_p$.

The dimensionality units Length, or $L$, or Time, or $T$, are also syntactic and are devoid of any semantic meaning.

As both $\vec{c} = k_m(m)$ and $\vec{c} = k_s(s)$ are orthogonal to each other, dimensional analysis allows the relationship between the two measurement units to be defined as follows:

$$k_m(m)/k_s(s) = k_m/m = k_s/s = k_v(m/s)$$

Unlike the units of measurements $m$ and $s$ the assignation of a numeric value to $m/s$ is not arbitrary, but rather must be calculated from the components $k_m$ and $k_s$. Since $k_m(m) = 1.16162 \times 10^{-35} \, (m)$ and $k_s(s) = 5.39124 \times 10^{-44} \, (s) \text{ then } k_v(m/s) = \ell_p/t_p = 2.99792 \times 10^8 \, (m/s)$.

The dimensionless displacement ratio $R_D$ can now be defined as $R_D = \beta(n)\vec{c}/n\vec{c} = x/L_0$. Since $n\vec{c} = \|L_0\| = \|\vec{r}\|$ it is possible, indeed advantageous, to re-define $R_D = x/\tau$. The dimensionless displacement ratio $R_D$ may now be associated with the dimension $L/T$. Label the term $R_D(m/s)$ the velocity or $v$. Let the maximum value of $v$, $R_D = \beta(n)/n = 1$, be called $c$, where $c = k_v(m/s) * 1$.

Let the displacement in the Z dimension be measured in units called $kg$ and let the dimensionality of a displacement in the Z dimension be called Mass or $M$. Unlike time or length, there is no requirement that displacements in the Z dimension be integer multiples of $\vec{c}$. The value of $k_{kg}(kg)$ is a derived quantity. However, let the unit distance of 1 * $\vec{c}$, be called a Planck mass, $m_t$. It should also be noted that the value of mass changes with velocity, and that the value of mass is always a minimum when the velocity is 0. Call this value of minimum mass the rest mass.

It is of prime importance to emphasize that the distance between the origin, coordinate $0, 0, 0$, and the system state, coordinate $(\beta(n), i\beta(n), z)$ must be an integer multiple of $\vec{c}$. This is the very definition of the metric axiom.
The dimensionality of the angle between the origin and the coordinate \((\beta(n),i\beta(n),z)\) is \(ML/T\). The distance between these two coordinates is of prime importance. There must exist a magnitude as well as an angle and the magnitude must be an integer multiple of \(c\). The dimensionality of the vector that connects the origin and the coordinate \((\beta(n),i\beta(n),z)\) is \(ML^2/T\). It is the length, or magnitude, of this vector that must be an integer multiple of \(c\). Label this vector the **metric vector**. Let the value of the magnitude of the unit vector, \(1/c\), be called a reduced Planck’s constant, \(h\), where \(h = k_B(h) = 1.05457 \times 10^{-34} (m^2kg/s)\). Similar to values of Planck time and Planck length the value of this numeric assignation is arbitrary.

The invariant metric \(c \times \pi\) will be called **Planck’s constant** or \(h\), where \(h = 2\pi h\).

5 Rewrite the Equations and Relation Using Units of Measurement

It is now possible rewrite the general, dimensionally independent, equations, relations, and constants using the defined units of measurement.

5.1 The System S Analog of the Lorentz Transformation

For a given displacement ratio \(R_D\), the displacement in the Y dimension:

\[
y = n \bar{c} \sqrt{1 - (R_D)^2}
\]

Is rewritten as:

\[
t = \tau \sqrt{1 - v^2}
\]

However, the dimensionality of left hand side of the equation \((L)\) does not match the right hand side of the equation \((L^2/T)\). To rectify this dimensional inconsistency the equation must be rewritten as:

\[
t = \tau \sqrt{1 - v^2/c^2} \quad (5)
\]

5.2 The System S Analog of the Einstein Field Equations

The curvature of the 3-manifold may be described by the equation.

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \alpha T_{\mu\nu} \quad (6)
\]

Where \(\alpha\) is a scalar, \(R_{\mu\nu}\) is the Ricci tensor, \(R\) is the Riemann curvature scalar, and \(g_{\mu\nu}\) is the metric tensor. The only change engendered by dimensional analysis of the system \(S\) is that \(\tau_{\mu\nu}\) is now rewritten as \(T_{\mu\nu}\). The tensor \(T_{\mu\nu}\) is labeled the energy-momentum-stress tensor.

5.3 The System S Analog of the Uncertainty Principle

It was previously shown that when \(R_D\) is small then \(\omega \approx \theta\) and the equation \(\Delta r \Delta \theta \geq \bar{c} \times \pi\) is approximated in Cartesian coordinates as \(\Delta x \Delta R_D \geq \bar{c}/2\). Factoring in dimensionality the relation can be rewritten as \(\Delta x \Delta v \geq \bar{c}/2\).

This relation needs to be extended into the Z dimension, now called the mass dimension. The expression that describes the covariant component \(\varphi\) of the momentum tensor \(P_{\mu\nu}\) \((\varphi = \arcsin(z/n\bar{c})\), is now written as \(\varphi = \arcsin(m/L_0)\) where \(m\) is mass and \(L_0 = n \bar{c}\) represents spatial displacement. The gradient \(\varphi\) has a value very close to 0 for systems with a small mass and a large spatial separation. In these instances the distance between the coordinates \((0,0,0))\) and the system state, coordinate \((\beta(n),i\beta(n),m)\) closely approximates the distance between the coordinates.
(0, 0, 0) and the coordinate \((β(n), iβ(n), 0)\). This relationship in turn allows a momentum vector \(p = mv\) to approximate the momentum tensor \(p_{ωφ}\) where the gradient \(ω\) corresponds to a velocity and the gradient \(φ\) corresponds to a system \(S\) analog of a gravitational field. Given that the gradient \(φ\) is small, a weak gravitational field, the momentum vector \(p = mv\) may be used in place of the momentum tensor \(p_{ωφ}\). This allows the metric axiom to be approximated by the relation:

\[
\Delta x \Delta p \geq \hbar /2 \tag{7}
\]

When the gradient \(φ\) is large, that is in a strong gravitational field, the momentum does not behave as a vector and so the analog of the uncertainty principle must be reformulated as a tensor equation. The proofs of these assertions and equations lie outside the scope of this paper.

It is easy to show that a displacement in the mass dimension is not defined when \(ν = c\) and also that for all values where \(ν < c\) there must exist a positive displacement in the mass-dimension.

When \(m/L_0 > 1\) the system becomes degenerate. Let \(m = 1 * c\) for the system \(S(1)\), a system of one event and one Planck mass. In this instance the metric axiom demands that the displacements in both the time and length dimensions must be 0 as the only allowable state for the system is \((0, 0, m_p)\). In the realm of physics these types of systems are referred to as black holes. The system described above represents the smallest possible black hole. If \(m > 1 * c\) the system \(S(1)\) is not associated with any allowable states.

Finally, a more detailed analysis shows that the analog of the field equations as written are also incorrect and must be reformulated to account for the notion of momentum as a rank-2 tensor rather than a vector. The existing equations will break down in the vicinity of the origin, a black hole. The alternative version of the field equations re-defines the energy-momentum-stress tensor as \(T_{ωφ}\), where \(ω\) corresponds to a velocity and \(φ\) corresponds to a system \(S\) analog of a gravitational field. These interpretations of \(φ\) are valid for the system \(S\) but may not be applicable to physics.

6 Conclusion

It has been demonstrated that some theorems in the system \(S\) look and behave in a manner very similar to the equations and relations of traditional physics. Of particular note is that the analogs of both the uncertainty principle and the Einstein field equations are emergent from the axioms. A mathematical bridge has been developed may describe a direct relationship between quantum mechanics and general relativity. However, it is important to note that these theorems are devoid of any semantic content. If the mathematics of these theorems is found to be valid then it may be possible to use the system \(S\) as the basis of a formal model of physics. This in turn would lead to robust discussion over the relationship between the formal system \(S\) and physics.

The most obvious question to arise from this discussion is whether the system \(S\) argues that the foundation of physics is inherently non-deterministic. To argue that physics is, at its core, a stochastic process represents a significant paradigm shift.

Perhaps the most prominent inconsistency between the theorems of system \(S\) and traditional physics lies in the treatment of momentum. Allowing momentum to be treated as a tensor rather than a vector enables the derivation of a consistent model of quantum gravity and also creates a mathematical framework for a version of physics that is generally covariant.

Ultimately, the applicability of this formal system to the discipline of physics will be determined by empirical experimentation.