From Physicality to Mathematicality, to Informaticality, to Ontology, and Consciousness

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Abstract.
Extraordinary mathematicality of physics is also shown by dimensionlessness of Planck spacetime and mass. At the same time the Planck granularity of spacetime also shows that physics can be simulated by a binary computer. So physics is informational. But mathematics is not everything in physics, consciousness cannot be solely explained by mathematics, and free will also not. The author claims also that consciousness without free will does not exist. Quantum mechanics (QM) is not complete, because foundational principle is not yet known, because consciousness and Quantum gravity (QG) are not yet explained, and because agreement between measurement and calculation is not everything. QG as dimensionless theory is the foundation of QM and not oppositely. The free will and quantum randomness are similar unexplained phenomena. Even philosophy is important in physics, because what mathematics cannot describe in physics is ontology. And, intuition affects what is mainstream physics. Simplicity and clearness of physics and mathematics of physics are important not only for beginners, but also for the development of the fundamental physics. Uncertainty principle is so simple that maybe it could be derived without the use of wave functions. On the simplicity and clearness of fundamental physics it can be done a lot. The reason that QG does not yet exist is the lack of knowledge about spacetime as background. QM and special relativity with less background are on a way. It is necessary to develop such QM that it will be compatible with general relativity, and that will be based on simple postulates. Mathematics, informatics, physics, and philosophy are languages, thus they need protocols and protocols are also mathematics. Naturalism seems a correct way toward foundations of physics, but deep meaning of Ockham razor in fundamental physics is expected by the author.

1. Introduction

The influence of mathematics in physics is an important issue. This is also connected by Feynman’s conviction that philosophy is not usable in physics. Some argue that mathematics is everything in physics, [2, 3, 4]. They also argue that it cannot be seen where non-mathematics can have any impact in the physical world. By contrast, others argue that mathematics is only an aid in physics, but it does not have a deeper meaning. They say that mathematics is only what can be defined. Therefore they say also that some of the deeper ontological questions of physics (philosophy), such as what is space, what is time, cannot be answered by physics.

One view on the relationship between physics and mathematics is a frequent assertion that quantum mechanics (QM) is complete, that does not need a deeper interpretation, that does not have some deeper background, and that it is enough to do that measurements agree with calculations. But on the other hand:

- if this were true, we would not need such papers as of Brukner-Zeilinger, [5, 6].
- Hypothetically, let us assume that we can guess a simple formula for the fine structure constant $\alpha = \frac{1}{137.035999173(35)}$, and let us assume that increasingly accurate measurements of $\alpha$ will be more and more in agreement with the formula. In that case it can be asked whether the model for the formula is really not needed. However, such formula does need a model, because the agreements between a calculation and measurements do not explain everything. Likewise, we also need a model for QM.

† Prolonged version of this essay is [1].
• Why do we have several interpretations of QM, although only one interpretation is enough? Several interpretations could mean several alternative backgrounds, including mathematical ones, rather than that a background would not exist.

• The foundation of QM is a theory of quantum gravity (QG). This, however, is not known, but it is important that it is dimensionless. So QM is not complete in itself.[§] Spacetime is some sort of foundation of QM. QG can tell more, how it is with QM.

• A possible candidate for the interpretation of consciousness is quantum consciousness. If this is true, QM is not conclusively explained if consciousness is not explained.

The influence of mathematics in physics can be imagined as the simulation of physical reality, for example, in the computer or in the movie Matrix.[7] Mathematics is what can be simulated with software, hardware is analogy for physics. It is generally expected that the proportion of physics itself is very small. Thus it should be that the impact of this hardware is as small as possible. So this thought experiment would be such that a computer would be as primitive as possible. In the movie Matrix, as well as in the continuation of the paper, consciousness of the observer of this simulation will be also included in the hardware. So, in essence, beside the philosophy, consciousness can also be added to this non-consensus between physics and mathematics.

The non-consensus, how mathematics affects to physics, is also in the fact that these things are estimated using intuition. For example, estimation by intuition is used also in a game of chess. A chess player calculates forward to certain moves, but after this he estimates positions by intuition. According to intuition he also chooses which moves he will calculate and which positions he will estimate. Better chess players better estimate positions, and better choose, which moves they will calculate. Their estimations are useful, but they are not 100%. For example, later when computers finally calculated some of well-known positions, which were estimated by chess players, they proved that sometimes their estimations were wrong. Even in physics, we use intuition to speculate about physical theories, or what is worthwhile to be read, or what can be allowed to be published. A lot of opinions about physical theories diverge just because of intuition or feeling. Thus, the cause of discrepancy between various views is many times in intuition, not in mathematics, an example is the above Feynman’s conviction.

At all success of mathematics in physics, all except mathematics is rejected in physics, because mathematics achieved the greater part in physics till today. But philosophy has also achieved a lot, for example, postulates of special relativity (SR), and general relativity (GR), are effects of philosophy. Besides, the basic question ”what consciousness is” has not been answered neither by physics, nor by philosophy, intuition, neurology, mathematics, and nor by simulations of neural networks, see fig. 1.1. Of course, these ones are the right ways to answer this question.

Thus, a fundamental physical theory may be composed of five elements: physics, mathematics, philosophy, consciousness and intuition. The argument closest to the author is that mathematics has a profound impact in physics, but it is not 100%. This is similarly as a cobweb. A cobweb (a synonym for the logical structure) can be large, but it must be fixed somewhere, fig. 1.2. Another more mathematical symbolism is [10, Fig.1b].

§ This does not mean advocacy of hidden variables of Bohm theory.
Agreements between calculations and measurements also have a profound impact in physics, but it is not 100%. When the theory of everything (TOE) will be known, however, it will be possible to say how this TOE's mathematics and ontology will be coexistent with itself, and how will differ from self-coexistence of other mathematical systems. Due to the unavailability of possible measurements that would show TOE, we should also develop mathematical and philosophical self-consistent theory of TOE before any measurements happen. But it can happen that we will be sure about its correctness despite of this. Such self-consistent theory will maybe help to find measurements that can confirm such theory in real time, for instance, measurements of the predicted masses of the elementary particles.

2. Dimensionless nature of physics

When Planck defined his constant $\hbar$, and then defined his time $t_{pl}$, the distance $l_{pl}$, and the mass $m_{pl}$, he also reached dimensionless nature of physics and thus mathematics was approached to physics. Namely, thus the masses of the particles can be described dimensionlessly, $\mu_i^2 = m_i^2 G/(\hbar c)$. At this, $m_i$s are masses of the various elementary particles, $G$ is the gravitational constant, and $c$ is the speed of light. If we imagine a universe where $c$ is hundred times larger and $m_i$s are ten times larger, the laws of physics would be exactly the same as in our universe. Thus these constants $\mu_i$s mean coupling between distance, time, and matter.

If this coupling would not have existed, quantities length, time, and mass had been independent among themselves, and had been indescribable with mathematics. Thus, they would be only physical quantities, not mathematical ones. But in the reality, mathematics describes them dimensionlessly. As an illustrative example, this coupling also disagrees with the claim that length is a physical quantity, but time is not. Planck also showed with granularity of spacetime that there is finiteness of information in physics. (This is consistent with the Zeilinger’s finding [5, 6].) Later it was discovered that the universe is, the most probably, finite. Again, this is a condition that physics is mathematical, or informational. When physics deals with finite size and the finite calculations, then this is just the right informatics [5, 6, 15]. Mathematical continuum is not informatical. Also, the above-mentioned physical simulations can be done only by a computer with finitely small steps. Computers with infinitely small steps have not yet been found, and physical simulations which need infinite calculation steps have not yet been found too. What is more, it turns out that the logic in physics is more fundamental than mathematics in physics, because binary computers are based only on logic.

The question is whether infinity exists at all. One mathematician even said to the author that infinity does not exist. Let us look whether a body can be accelerated with such speed that it reaches $c$. As far as it

\[ \text{That are a consciousness theory, QG, masses of elementary particles, the fine structure constant, big bang physics, etc.} \]

\[ \text{About this it is a lot written by Duff, [11, 12], for instance, (1) that variation of dimensionful quantities is unphysical if dimensionless quantities do not vary. He also claims that (2) dimensionful quantities are always in surplus according to dimensionless ones, and that (3) constants } \]

\[ G, c, \text{ and } \hbar \text{ do not exist physically. But, the author disagrees with the last two claims. Zee also wrote about this that, } c, \text{ and } \hbar \text{ are not so much fundamental constants as conversion factors.} \]
approaches \( c \), it is always in some inertial system, where it is stationary. So it is never more close to \( c \) and infinite mass.

So this means mathematicality or informaticality of physics. Mathematics and informatics are both effective languages. Languages need protocols, what are dimensionlessness and finiteness. Mathematics is a language that we can talk easier about physics, about philosophy, even about ontology. All other than mathematics is rejected also because debates at mathematics of physics are much clearer to show who is right and who is not. Other non-mathematical debates can never end. But, a better protocol can be made also for the discussion that it can be clearer what is correct and what not about a philosophy debate. But, this protocol is also some type of mathematics.

For the physical world around us, we can say that it is a thing of ontology, and a thing of mathematics. With the above Planck’s findings of dimensionless nature of physics, ontology of physics decreases and mathematicality increases. Due to the Planck’s finiteness of physics, mathematicality, or more precisely informaticality, of physics is also growing. Because of such findings, many physicists assume that everything is just mathematics, nothing else, [2, 3].

Although the theory of QG does not yet exist, the Planck’s dimensionlessness and finiteness are steps toward it.

With Planck, physics also partially answered to some basic philosophical questions about space and time, therefore, it cannot be said that physics does not answer on such questions.

With the above Planck dimensionless physics it can be confirmed that the mathematics also defines physics, because physics becomes dimensionless. Thus, it is not necessary to say that only mathematics adjusts to physics, but it is also vice versa.

But, even physics defines mathematics. For example, the Pythagorean theorem is a consequence of Euclidean space, and Euclidean space is the result of physics. If a trolley is moving in a rocket (direction \( x \)) transversally in the direction of the rocket (\( y \)), its kinetic energy is equal to \( W = m(v_y^2 + v_x^2)/2 \) where \( v_y \) is speed of the rocket, \( v_x \) is the speed of the trolley with respect to the rocket, and \( m \) is the mass of the trolley. Let us look at the picture 2.1. Orthogonality thus occurs as independent of Euclidean space. Admittedly, it is not entirely clear to the author if really all derivations of Pythagorean theorem (implicitly) involve this energy theorem.

![Figure 2.1. We have the rocket moving in the direction \( y \) with speed \( v_y \). The trolley on the rocket is moving transversally to direction of the rocket with speed \( v_x \). Velocity of the trolley according to an external observer can be calculated with Pythagorean theorem. Sum of kinetic energy of a rest trolley in the rocket, \( mv_y^2/2 \), and of energy of local movement of this trolley in rocket, \( mv_x^2/2 \), also agrees with Pythagorean theorem.](image)

We can ask ourselves, how to build mathematics if we and physical time do not exist. Thus, mathematics is a consequence of physics. This is close to Smolin [16], but not in all.

+ A new word: As "physicality" means generally physical view on world, "informaticality" means informatical view on physical world.

* Admittedly, it is not known how to use \( \mu \)s of elementary particles, because they are much smaller than 1. But, QG is still unknown, so it is possible that such small numbers for small black holes exist at the expense of something else. Of course, the theory of the Higgs boson will tell us if this is appropriate.
3. Consciousness

Consciousness still needs to be connected with physics and neuroscience in several aspects, our knowledge about it until now is not enough. Consciousness is not yet described by a physical quantity, such as the length is. It is still less described with a dimensionless quantity such as $\mu s$. That the relationship between consciousness and physics somehow exists, it is indicated by quantum measurements, because it seems like that a quantum measurement is triggered by consciousness. A claim should be added that the only reality that exists is consciousness. The author has added to this basis the article [17, 18], which says that the physical world is based upon panpsychism, i.e. on a primitive units of consciousness. If consciousness is everywhere, the physical world is based upon mathematics, upon physics, which will be increasingly reduced to mathematics with new knowledge, and upon consciousness, which is probably the only reality in addition to mathematics. So probably, it is based solely on mathematics and consciousness. We can imagine two scenarios which exclude each other:

- that consciousness arises from processes in the brain, and
- that matter (physical world) arises from processes in the consciousness, panpsychism.

It is hard to imagine the first scenario, because in spite of all processes (mathematics) we cannot know how a new stuff arises, that is consciousness. Thus another scenario left, which is not contradictory intuitively.

Consciousness is based on qualia. We can imagine how three basic colors mean very different feelings. Now it is difficult to imagine that these three qualia have the same origin, such as mass, time and length have the same origin. But in the spirit of Ockham razor in physics, they should also be brought to the same base. So, according to the author’s opinion, everything is just panpsychism, and this is as far as possible "monochromatic", then it is mathematics, but physical objectivity is this objectivity, which is consequence of panpsychism. Thus in fig. 1.2 the fence means panpsychism, and the cobweb means mathematics.

One of general objections is that consciousness is a thing of macroscopic world, [19]. However, consciousness should be so reduced as physics is, to see that the basic “bit” of consciousness is memory. And then we can ask ourselves whether consciousness is in a unicellular organism, or even in collapse of measurement of one qubit. Namely, as it is relatively whether the privileged inertial system is this one or the other one in SR, so it is relatively if memories in one or another person mean one or another ego. Consciousness and ego are defined by memories of qualia.

It is good to mention here the physical principle that supports the quantum consciousness. For someone who looks from the outside, the difference between a philosophical zombie (p-zombie) and a conscious person should exist, so that it can be distinguished that consciousness responds differently than an automatic programmed zombie. This is called the "Turing test". How it can be performed is evident in [1, p. 8]. It shows that consciousness needs still free will, similarly as physics needs movement.

Sheldrake has a similar standpoint about consciousness, [20, 21], thus independent thinking converge toward the same conclusions. Even Koch advocates panpsychism [22].

Today it is said at every mentioning of connection of quantum measurement and consciousness that this is unscientifically. But, panpsychism suppresses illogicalities at these models. Maybe preferring of panpsychism seems strange, but it is still more strange that it is claimed that consciousness can arise from matter, that free will does not exist, and, still worse, that consciousness does not exist. Otherwise, panpsychism still ever includes physicalism and reductionism, as other approaches in physics include them, with exception that materialism is not included.

4. Simplicity

In essence, simplicity and clearness are two necessary and interweaved approaches.

So far existing physics is simple on same way. It can be that models are not simple, but postulates are. Simplicity in SR are Einstein’s postulates, simplicity in GR is the principle of equivalence. QM

‡ The argument of many, also of David Chalmers.

†† It could also be imagined a larger number of primary colors, such as 12, as Mantis shrimps see them. It is not sure that they see so much colors also in the brain, not only in the eyes, but nevertheless, let us assume this. Checking of number of color qualia of Mantis shrimp could verify important background information about qualia. If this number is only three, this is probably limitation.
suffers from a clear foundational principle, but granularity, granularity, and dimensionlessness of physics mean simplicity. UP also means simplicity, because it is all-embracing and necessary for QM, and it is described with a simple formula.

Symmetry of physics is also connected with simplicity. Panpsychism is also a way that connection of matter and consciousness is made simpler. Linearity also means simplicity.

It is said many times for QM that only agreements between measurements and calculations are important. But logical structure of the formulae is also important, and simplicity of the formulae is also achieved.

Simple approaches can also be achieved by seemingly wrong models, for instance, with the Bohr model of an atom. Although it does not give enough precise result of calculation, this is approach in the right directions and can be called lower approximation. It is a good example that "not precise and simple" is sometimes better than "precise and complicated". This can improve comprehension at beginners and at others also. It is also similarly with the Heisenberg microscope.

Simplicity of physics will also be improved if some short and clear formulae would be derived in less steps. For instance, until now, a visually appropriate explanation of UP is evident in endnote. This derivation is short, but it is wished that it should be still shorter. It would be clearer if UP can be derived without the use of wave functions. Because, UP is more generally valid, it is connected with information, and Gaussian curve is very basic in statistical calculations. Derivation of UP without wave functions is still unknown, but it seems that it is on a way because of [5].

The author also gave some examples how these direct derivations should look like, as example of derivations of SR, [23].

A question for so far existing theories of QG appears, are they correct, because they are not simple? It would be good, that such theory would be predictable and that would agree with physical intuition which is in us. It would also be expected that QG theory would predict at least one postulate ...

5. Clearness

The second thing connected with simplicity of physics is clearness of physics. Physicists are too much satisfied only with correctness of derivations. But, all these derivations can be repeated in such way that they become clear. And, it is useful that derivations are made from various aspects. For instance, interpretation of SR, [23], is an example of wish how to improve presentation also at QM.

Clear presentations of physical formulae are important for beginners, but also for physicists, developers of QG. For instance, Lorentz contraction (LC) is usually presented with a system of equations, but this does not give a proper mental picture of derivation of LC. Derivation of LC in [23] gives better mental picture. It is important because it gives a new aspect about LC.

Bizarreness of some new theories is disturbing for people with orthodox understanding of physics, thus this is also a reason that many of new theories are not developed, although physics should be viewed from new aspects and from new postulates. For instance, physicists do not like relativistic mass in explanation of SR, [25], similarly as chess players do not like uncommon chess positions.

Mathematics can be very effective to provide clearness of physics. For instance, imaginary distance between two events in space of Minkowski means that speed between them should be larger than c, and thus is impossible. The formulae of SR also show causality. SR also tells that the essence is resting in inertial systems. Minkowski space distance also shows that time is different as distance. Imaginary nature of wave functions tells also that they cannot be seen alone, but only as probability density. Mathematically it is possible to imagine something, although it does not exist physically, for instance imaginary numbers or space out of our universe. This helps at visualization of real physical world.

Clearness will be achieved also by derivation of UP without use of wave functions.

Examples of better clearness at GR are [26, 27], and in QM are [28, 29, 30]. These are examples of approaches which improve visualization and clearness.

The approach to clearness is also to understand physical facts with our physical feeling of outer world. For instance, if we compress one lump of clay, we feel resistance at this. Thus a lump of clay contains energy

† Zeilinger: "I suggest that the message is that a generally accepted foundational principle for quantum mechanics has not yet been identified." [5].

‡ Instead of this it is sometimes valid at other complementary quantities.
and confirms existence of Einstein’s equation $E = mc^2$. If some matter cannot give response on any possible way then it does not exist. A lump of clay gives also consciousness of feeling of its response, thus this is also a conscious response.

As further, because of SR, rigid bodies do not exist. Thus force travels with finite speed, this means it travels like a wave. This gives feeling, why waves are so important in QM. (One speculation is also that digits in numbers are also cyclic, as consequence of logic which build up numbers and then physics.)

Feeling for physics begins soon after birth, when a baby sees how something falls from a table. This is above all feeling for Newtonian physics, but is not feeling for SR, GR, and QM. Feeling for otherwise strange QM could be obtained if we as children could play computer games about quantum world. Such principle could clarify, how QM is really strange and how it is only a consequence of our bad feeling of it. Because the author does not believe in completely wrong intuition of humans, strangenesses in these theories should be explained with Newtonian physics, and with other logic and mathematic, which we learned from birth.

Other fundamental physics, where intuition is obvious and it can help at comprehension, are dimensionlessness of physics, relativity of inertial systems, equivalence principle, awareness that consciousness is not an illusion, existence of free will, awareness that consciousness cannot be built up from the processes in matter, and that QM needs more than agreement between measurement and prediction.

Some author’s examples of better visualizations of physics are evident in endnotes, and Newton’s physics versus QM, SR, and GR is like to change currency to another currency in one country. At first people calculate in the old currency and then gradually begin to calculate only in the new currency.

But, this clearness is also connected with a new view on postulates, and it opens new calculations, and new theories.

### 6. Extrapolations

Another important issue of mathematics in physics is also extrapolation. For example, will QG be similar to current theories of SR, GR, QM, and quantum field theory (QFT), or it will have some new elements? This extrapolation is complicated by the fact that QM and GR are different, QM has a fixed background spacetime, but GR does not have this background.

It would be good to achieve these common initial conditions that QM and GR could be extrapolated into the common QG. Namely, it is difficult to say how to merge GR and QM if they are already different in principle. But, because it is possible to develop SR with less background, it is maybe possible to develop QM and QFT with less background. In it is also showed how to derive QM with less background. This theory is more linked to information and less to spacetime.

One example of the merging, which ignores the different background, is also a rocket on photonic propulsion. Let us assume that gravity cannot be less random than the propulsion, which gives force felt by the rocket. Thus we can guess with the principle of equivalence in quantum regime ...

Extrapolation of Kitada also exists. Although, probably it does not merge QM and GR correctly, it can show problems at the merging from a distinct aspect, and it is possible to guess on a different way how to proceed. Thus, this is also an approximation.

According to extrapolations of GR and QM it is a vital problem whether spacetime exists if absolutely no rest matter exists. Some people claim that it exists, but the author claims that it does not exist.

It is important not only that QG is extrapolation of SR, QM, and GR, but these three theories are extrapolations of QG toward other size regimes of energies and spacetime. It is claimed that QM is a perfect theory where nothing was found what can disprove it. But it is necessary to respect that QM is only an extrapolation of QG, which is still unknown. At the same time, the foundation should be a dimensionless theory, what is QG.

Because QG is occupied with fundamental building blocks of physics it could be said a seemingly wrong sentence that quantum consciousness is unknown, because QG is unknown. A possible objection can be that quantum consciousness operates in different size regime than QG. But, despite of this, consciousness is one of the most basic concepts of physics, one example is also panpsychism. Thus, we should to know what spacetime and matter are, that we can merge them correctly with consciousness. QG will also tell...
more about quantum randomness, what can be connected with free will. QG means also more mathematical knowledge about physics, and this means to be closer what is consciousness, which is probably all the rest. It is also necessary to know if space and time give such informaticality as matter. Informaticality means to be closer to consciousness. Thus, QG is needed, Newtonian physics is not enough.

An extrapolation is also a claim of neuroscientists that Libet experiment, \cite{Libet1985}, gives that free will does not exist. And, that consciousness is only in the brains, thus that panpsychism does not exist.

One important element in physics and extrapolations is also linearity. This means, if something is nonlinear, it has some new elements, it has some new impact. For instance, if bricks are put one on another, the total height of a pile will be sum of the heights of the single bricks. But, let us assume that the bricks are contracted under the pile. In this case we obtain this nonlinearity, fig. 6.1. Such linearity and nonlinearity have impact in physics on various ways.

![Figure 6.1. The bricks in the pile, right, shrunk because of weight, as an example for unlinearity.](image)

7. Quantum randomness

As one of the most fundamental questions is quantum randomness. In \cite{17,18} it is tried to answer that quantum randomness is a similar mechanism as our free will, and that superposition of quantum states means possibility of decision, and that quantum randomness is a free decision in panpsychism. The most known example of quantum superposition is in endnote \cite{D}. Both mechanisms, free will and quantum randomness are primary, without any other more basic elements in physics. Quantum randomness is even also called the free will, \cite{38}. If quantum randomness and free will are explained together, unknown quantities in physics are reduced.

But so it seems strange to us, because free will is not absolutely random, but it is dependent from our decisions. This was also tried to be explained in \cite{17,18}. One dilemma is, whether free will is absolutely random, would we decide and move absolutely randomly. Movements of a hand would be absolutely random. But, they are not. The author suspects that this is such nonlinearity of quantum physics, that quantum physics is different on the level of the human brain as on the level of atomic world, where quantum events are not connected. Of course, explanation of this needs some more precise model and measurements. Thus, if we have some group of completely independent and free people, their decisions for hands movements will be completely unconnected, similarly as quantum randomness in the level of atoms is.
ENDNOTES

A. Uncertainty principle explanation

Figure A.1 The essence of derivation of UP is shown. In (a) we can see that wave function in momentum space is Fourier transformation of the wave function in physical space and vice versa. In [39, Point 8]) it is also evident, how widening of one curve means contraction of another and vice versa. (b) If the envelope of the first wave function has Gaussian shape (c) then the envelope of the second wave function has Gaussian shape too. But, the product of widths of both envelopes, $4\Delta p\Delta x$, stays the same. (We should also be aware that $|\Psi|^2$s are also Gaussian curves, and they are a last step toward UP.)

B. Precession

If one gyroscope is hung on a cord and is rotating with its axis in a horizontal direction, then this axis will rotate in horizontal plane, or it will precess. This is drawn in the left side of fig. B a. Because the result of this phenomenon is counterintuitive, this is explained visually in the of fig. B b.

Figure B.1 In fig., a, it can be imagined, how precession is obtained if a gyroscope is hanged on a rope on the last end of the axis. $I\Omega \times p = L\Omega = \Delta L/\Delta t = F \times r = M$, where $L = I \times p$, $I$ is moment of inertia, $L$ is angular momentum of wheel, $p$ is angular frequency of rotation of wheel, $M = m g \times r$ is torque on a gyroscope, $r$ is length from the last end of the axis to the wheel, $F = m g$ is force of weight of wheel, and $\Omega$ is angular frequency of precession. Thus, $\Omega = m g r/(I p)$. If the gyroscope is rotating anticlockwise, the axis will be rotating anticlockwise according to the horizontal plane. Figure a is from [40]. Because
it is not easy to comprehend why axis does not fall down, but it is rotating aside, in the right side, b, it is visualized, why torque down means rotation aside. The rotation of wheel is shown as distributed among four momentum vectors. Because the torque forces wheel down, the axis falls down a little, and two of four vectors change a little, as evident in the cube extremely down. Because of conservation of momentum, reaction on this change arises, thus two vectors of momentum arise in \( z \) and \( -z \) directions, which are seen in green color. These two vectors mean that axis will rotate to the right, thus it will rotate anticlockwise.

Another explanation instead of the above b example is in [41, Point 5]. A video is shown [42], which shows precession. One interesting application of precession is also [43].

**C. Visualization of one logical statement**

One example of a logical statement is computer source code, when we ask whether intervals 1 and 2 have some intersection. This can be written as:
"if \((\text{min}_1 \leq \text{max}_2) \) and \((\text{min}_2 \leq \text{max}_1)\)."
At this, both "\( \text{min} \)" mean minima of these intervals and both "\( \text{max} \)" mean maxima of these intervals. This can be the easiest way to visualize and to remember this statement, as it is shown in the quadrilaterals in fig. [C] This can be easier to imagine than the above logical statement. Symmetry of this sentence is also illustrative if the quadrilateral is mirrored over the horizontal or the vertical axis.

**Figure C.1** In both instances both diagonals start at the left (green) sides of the quadrilaterals and their arrows end at the right (green) sides of the quadrilaterals. Fig. a: If intersection (black lines) of both horizontal (blue) lines exists at \( x \) coordinates, then directions of both diagonals are oriented toward the right side of the fig. a. Fig. b: If intersection of horizontal lines does not exist, then one diagonal is not oriented toward the right side of the fig. b, the dashed one.

**D. An example of the collapse of the wave function**

**Figure D.1** The left side of the figure shows superposition state of electron spin. In the right side, after the measurement, the collapse of wave function happens, either in spin up or in spin down. A measurement behaves like free will decision.