Abstract. The success of quantum theory in describing the particle forces has been assumed to imply that quantum theory is fundamental. This assumption has been integral to the search for a unified physics theory, but what if it is wrong? Questioning why we needed quantum theory in the first place is directly answered by experiments revealing electrons to possess a wave property that cannot be derived in classical physics. What if this is exactly as it sounds?

1. Introduction

There has been a history of warnings, first in philosophy and then science, about generalising from limited observations to asserting a “universal truth” [1]. Generalising the success of quantum theory describing particles under experimental conditions to it being true under all possible conditions, is such a questionable induction. As there is currently no experimental evidence that gravity is quantised, the assertion that gravity must be quantised is solely based upon induction from particle forces to all forces, which may not be true. The difficulty in finding a unified theory with quantum gravity makes it worthwhile to consider the possibility that quantum theory may not fundamental.

It might be expected that some new theory for quantum effects would be sought, but instead we will proceed by considering an underlying assumption of science that is so fundamental it is almost beyond questioning, that observations will always be derivable in an accurate scientific theory. This belief has persisted despite proof [2] that not everything within the scope of a theory can necessarily be derived. The difficulty with Gödel's incompleteness theorem is that its meaning has often been misunderstood, and has instead been used to argue that some things are beyond science. These sorts of arguments completely miss the point: there can exist statements within some types of theory that are true, but cannot be derived in the theory. Every term in a classical physics theory will be experimentally measurable in some way, and so can be said to be physically real. As this would also include the terms in a true statement that couldn't be derived in theory, Gödel's proof implies that it is possible for there to exist an observation expressed in physically-real terms that cannot be derived in classical physics. Science has implicitly assumed that this never really occurs, but the statement in classical physics that a particle is observed to possess a wave property is exactly of this form.

In classical physics, there is a classification divide between the motion of individual particles, and wave motion propagating through collections of particles, such as sound waves, or waves in continuous fields like light. The experimental observation of electrons passing through slits generating the wave characteristic of an interference pattern challenges this divide, although this is only really clear when the electron beam intensity is reduced to the point of a single electron passing through the slits at a time [3]. Each electron impacts the screen at one specific point, as for a particle, but the cumulative impact of electrons builds up an wave interference pattern showing that each electron travelled through the slits as a wave. There is no means in classical mechanics for a single particle to travel as a wave and so it is known with certainty that
the observed wave property cannot be derived in classical physics. And yet because classical physics contains both particles and waves, the statement that a particle has a wave property is within classical physics.

So in the physically-real terms of classical physics, we have an observably true statement that cannot be derived, just as in Gödel’s incompleteness theorem. The notion that just because an observation cannot be derived means it isn't science, is nonsense that conflicts with the origin of many science disciplines. Whenever you have a collection of observations of a physical system, it will be possible to find an empirical theory that relates these observations, this is after all how classical mechanics began with Galileo. However for a non-derivable wave property of a particle it is necessary to change the character of the terms in order to get such an empirical theory, and this is where it gets interesting.

2. Physically-real terms

Theories in the physical sciences are generally concerned with object systems where interactions cause the objects to change state e.g. $A \rightarrow B$ or the different object types react with each other e.g. $A + B \rightarrow C + D$. Even the notation of these examples relies on the foundational assumptions which define the character of classical physics. The $A, B$ are physically-real terms denoting objects which can be measured and classified into sets by type. The number of objects in these sets provides the definition of natural-numbers in classical physics theories, and gives the variables needed for Gödel’s incompleteness theorem to apply [4]. Object reactions denoting causal changes e.g. $A \rightarrow B$ give logical implication in physically-real terms within classical physics and also produce changes in object numbers that can give rise to the operations of arithmetic being in physically-real terms within a classical physics theory.

Using physically-real terms to denote objects is critical to the creation of a deductive theory that is logically consistent, where logical truth values in classical physics theories of objects are implicitly based upon the existence of an object state – True if the object state exists, and False if it doesn't – which is not so clearly the case in quantum theory. However in experimental terms, a particle is always measured to either exist or not, and so the apparent ambiguity of particle existence in quantum theory seems to only be in theory, not in the physically-real terms of experiments. The known consistency of physically-real classical physics is important for Gödel’s incompleteness theorem, because when the proof applies to a theory it cannot then be proven to possess the consistency required for the proof, and so the theory has to be known to be logically consistent beforehand.

Gödel’s proof can be summarised as applying to any consistent deductive theory with arithmetic over the natural-numbers, but it technically requires the theory to contain all possible number-theoretic functions over any number of natural-number valued variables [4]. This is not so demanding as it appears, as such functions can be created from 4 initial functions – successor, predecessor, zero and projection functions – by the repeated application of the function creation rules of substitution and recursion. Every classical physics theory of object state changes will contain these 4 initial functions, but special physical conditions are required for the function creation rules to be in physically-real terms within classical physics [5]: substitution requires a tree-like hierarchy of object reactions; and recursion requires the creation of a new countable entity that didn't previously exist in the theory. These conditions are only met in classical physics for a growing network of object reactions that include processes which implement arithmetic increases in the numbers of some objects.
However, Gödel’s proof demands every recursive number-theoretic function to be expressed within the theory, which can only be met in classical physics by an infinite network or an indefinitely growing network that can be infinite in theory, but not necessarily in practise. Although the latter is possible in biology [5], the classical physics of particles would require a single particle to start off an infinite sequence of particle reactions, only for the particle reaction network to collapse back down to the original single particle. As there is no obvious means in classical physics to have a finite physical network that is potentially infinite in theory, this situation could only arise in the context of an infinite network expansion about some object. Such an infinite particle reaction expansion is the mathematical form of the path integrals of quantum field theory, and the incompleteness conditions imply that if an analogous expansion arose for some reason within classical physics, the theory would be proven incomplete.

Assuming that this is true, and that the statement of a particle having a wave-property is an example of a true non-derivable statement, the first point to note is that there could be no hidden variable theory in classical physics that would able to derive this. The incompleteness proof is such that all possible natural-number variables are considered in the course of the proof, due to the unlimited recursion underlying the generation of every number-theoretic function within the theory. Conservation laws applying to the charges of particles mean that no real-number valued variables could be the cause of changes in particle numbers, and so additional variables of this character would have no effect on the incompleteness proof.

3. Non-physically-real terms

The character of the incompleteness proof is such that the wave property could only be an non-derivable property of the whole infinite network, but we are assuming that it is a property of the single particle that the expansion is about. This conflict between a single particle and its infinite network expansion is irresolvable in physically-real terms. However, these problems can be resolved by relaxing the insistence on physically-real terms and instead denoting particle numbers by a real-number valued term ψ that includes the wave property, whilst retaining the principles of space-time and particle symmetries from which the particle physics can be derived. This changes the situation because the incompleteness proof only applies to relevant theories over the natural-numbers, not over the real-numbers. So whereas the consistent theory of the network expansion about a particle is assumed to be incomplete, and is consequently such that no modification of the theory can make it both consistent and complete by including the wave property, a theory of the same form over real-number valued variables for particle numbers keeps the form of the classical physics, but can include the wave property and so be a complete theory. It could reasonably be expected that such mathematical slight of hand would come with a catch, and it does.

As particles are always experimentally measured to be countable, the real-number valued variable ψ for particle numbers is a non-physically-real term that does not directly correspond to experiment. For a modified theory with these non-physically-real terms to make predictions for the particle numbers of an experiment, some function must be added to recover the physically-real value, which must be of the form \( x = M(\psi^\dagger \psi) \) in order to remove the wave property from the prediction of particle number \( x \). This form of expression for an observed variable subject to local causation in Special Relativity gives the necessary conditions for the spin statistics theorems [6], where the term \( \psi \) must posses either spin \( \frac{1}{2} \) or 1 and have the associated statistics of fermions or bosons.
If $M$ could be deduced in classical physics, that would mean $M$ was of the form of a number-theoretic function, but if $M$ could be deduced then so could its inverse $M^{-1}$ [4]. This inverse could then be used to reverse the replacement of $x$ with $\psi$ to give a theory that supposedly retained the consistency of the original theory, but included the wave property and so was a complete theory. But Gödel’s proved that such a theory cannot be both consistent and complete, implying that the assumption of $M$ being derivable was wrong. Applying the same argument to a simple interpretation $x = I(\psi)$ gives the conclusion that the interpretation cannot be consistent.

One inconsistency is that $\psi$ simultaneously denotes particle and wave, which are on opposite sides of a classification divide in classical physics. Further inconsistency comes from the term $x$ being defined in theory by the cardinality of a set of particles, whereas the term $\psi$ doesn't reference a set. So the interpretation $x = I(\psi)$ relates a set term $x$ to a non-set term $\psi$, which inevitably gives an inconsistency when interpreted in the context of sets of particles.

This shows that many of the unusual characteristics of quantum theory can be derived from the assumption that a particle with a wave-property is a non-derivable statement in classical physics [5]. This would imply that quantum theory is not fundamental, as the defining properties of something fundamental shouldn’t be derivable.

4. Virtual-radiation

Confidence in this conclusion would be enhanced by finding the required infinite network expansion about some particle-like object in classical physics. The $N$-repeat control of a reaction process underlying any realisation of addition $n \to n + N$ in the physically-real terms of particle numbers would be expected to require an energy source to power it, and entropy considerations for object arithmetic arrive at the same conclusion. But the only experiments apparently indicating a non-particle source of energy are those confirming the Casimir effect of quantum theory [7, 8]. However, as all experimental results are by definition in physically-real terms, the Casimir effect results are expressed within classical physics, and so all might not be as it seems.

The standard explanation of the Casimir effect [9] is in terms of a vacuum state containing an infinite range of ground state radiation modes, some of which are excluded from the space between two metal plates, and this energy difference in vacuum states gives a force of attraction between the plates. However, the Casimir force between two metal plates can also be derived as a vacuum polarisation effect involving virtual-currents [10]. This begs the question, which of the two scenarios do experiments confirm? A vacuum ground state of radiation, or virtual-radiation in a vacuum polarisation region about a physical object?

Now the event horizon of a black hole is the surface where the radial component of the metric becomes infinite $g_{rr} = \infty$, and gives the same surface as defined by the time component $g_{tt} = 0$ in the Schwarzschild metric of a stationary black hole. As a black hole rotates faster, this second surface lifts off the event horizon and bulges out at the equator to give an ergo-region [11] where the sign of $g_{tt} < 0$ in normal space is reversed. The effect of this sign reversal on radiation that is massless in normal space, $C_1 = g_{\mu\nu}P^\mu P^\nu = -E^2 + p^2 = 0$, is to convert it into virtual-radiation $C_2 = E^2 + p^2 = -m^2$ with $m^2 < 0$ inside the ergo-region. So unlike the infinite radiation ground state of the vacuum, a region capable of containing virtual-radiation is known to exist about a physical object in classical physics.

The apparent problem is that the object is a black hole, which is predicted to emit a black
body spectrum of Hawking radiation with a temperature that is inversely proportional to the black hole radius [11]. So a particle sized black hole should be so hot that it rapidly emits all its mass as radiation and blinks out of existence. However, if the event horizon of the black hole carries a conserved charge that cannot be radiated away, there will be some limit to this emission of radiation. Kaluza-Klein theories [12] are classical physics theories extending General Relativity, where there are extra dimensions which are assumed to be closed and shrunk (compactified) down to the Planck length \( l_p \). Such compactified extra dimensions could be such that a rotating black hole bearing a conserved charge could shrink down to the Planck scale, but no further. For now, we will simply assume that this can be true.

In the Kerr metric of a rotating black hole with angular momentum \( j \) and mass \( m \), the radius of the event horizon is given by:

\[
r = \frac{Gm}{c^2} \pm \sqrt{\frac{G^2 m^2}{c^4} - \frac{j^2}{m^2 c^2}}
\]

The square root means that the radius will only have a real-number value for angular momentum \( j \leq \frac{Gm^2}{c} \). For the scenario where this angular momentum bound is reached and the event horizon radius is the Planck length, the black hole mass is the Planck mass \( m_p = l_p c^2 / G \), and the angular momentum \( j = \frac{Gm^2}{c} = m_p j / c \) is Planck's constant \( \hbar \). Now the standard wisdom is that the appearance of Planck's constant indicates the arrival of quantum theory, and if the conclusion of the previous section is true, this is indeed the case, because a wave expansion about this particle-like black hole gives the infinite network expansion of section 2.

### 5. Quantum field theory

Although such a black hole is assumed not to emit Hawking radiation, virtual-radiation in the ergo-region would have a mass reduction effect analogous to the Penrose process [13], and so its physical mass would be less than the Planck mass. The question is by how much is the mass reduced, because the masses of real particles are much less than the Planck mass. The obvious approach to adopt is a wave expansion for the virtual-radiation in the ergo-region. In Kaluza-Klein theory [12], the Hamiltonian for the classical physics theory with compactified dimensions involves gauge fields \( A \) and scalar fields \( \Phi \) as continuous fields, but the particle-like black holes will be classical physics objects that are countable as \( n \). The generic form of the Hamiltonian for the energy of a configuration of such particle-like blacks holes is:

\[
H = \int d^3 x |\det g_{\mu \nu}|^{1/2} \{ \overline{H}(A, \Phi) + \overline{H}(n, A) + \overline{H}(n, \Phi) \}
\]

The first term includes the energy of the gauge field strength and the corresponding energy for the scalar field. The next two terms generically denote interactions between the particle-like black holes and the gauge and scalar fields respectively. The wave expansion in the gauge fields \( A \) and scalar fields \( \Phi \) will be of the form:

\[
A_{\mu} = \int d\omega \int_k A_{\mu}^k e^{i(\omega t - k \cdot x)} dk^3 \quad \Phi = \int d\omega \int_k \Phi^k e^{i(\omega t - k \cdot x)} dk^3
\]
where the frequency integral is required because $\omega \neq |k|$ for virtual-radiation in the ergo-region. If the physical mass $m$ of the black hole is reduced by this virtual-radiation field to be much less than the Planck mass, the energy of individual wave modes in the expansion will be sufficient to create particle/anti-particle pairs for $E \geq 2m$. This introduces recursion into the wave expansion, as the expansion for the reduced mass $m$ will now include wave terms that give rise to particles with the mass $m$ being calculated. The same Hamiltonian expansion must then be recursively repeated for every particle/anti-particle created in the initial term, and this gives an infinite series expansion for the recursion:

$$H = \sum_{n=0}^{\infty} \int_{\omega_e}^{\omega_o} d\omega_0 \int_{\omega_1}^{\omega_0} d\omega_1 \ldots \int_{\omega_{n-1}}^{\omega_{n-2}} d\omega_n H_0(\omega_0)H_1(\omega_1)\ldots H_n(\omega_n)$$

where $\omega_e$ is the upper limit given by the dispersion relation for the virtual-radiation in the ergo-region. The sign reversal of $g_{\mu\nu}$ in the ergo-region also impacts space-time separations such that causal events involving the ergo-region can have space-like separations, which gives a distance equivalent to the mass $m^2 < 0$ in the ergo-region. Switching the integrals from phase-space to real-space, the time integrals will be subject to a corresponding ergo-region constraint, and so the recursive series expansion in terms of space-time integrals is:

$$H = \sum_{n=0}^{\infty} \int d^4x_0 \left| \det g_{\mu\nu} \right|^{1/2} \int d^4x_1 \left| \det g_{\mu\nu} \right|^{1/2} \ldots \int d^4x_n \left| \det g_{\mu\nu} \right|^{1/2} \bar{H}_0(x_0)\bar{H}_1(x_1)\ldots\bar{H}_n(x_n)$$

This is of the same form as the path integral expansion in quantum field theory [14], but here it is a recursive expansion for the mass reduction of a compactified black hole by virtual-radiation in the ergo-region. This recursive expansion can be expressed as:

$$H(n, A, \Phi) = \sum_{n=0}^{\infty} \sum_i T_n^i(n, A, \Phi)$$

where the variable $n$ gives the recursion depth, and the variable $i$ runs over the combinations of reaction sequences possible at that depth. Each term is of the form of a reaction sequence that starts with a single particle emitting virtual-radiation, and then both the particle and radiation take part in a reaction sequence that ends with the initial particle again. This is exactly of the form of the infinite network expansion of the incompleteness proof in section 2, such that the change in variables of section 3 gives quantum theory as an empirical theory that includes the observed non-derivable wave property of a particle.

However, the space-time integrals of the Hamiltonian expansion include the determinant of the Kerr metric $g_{\mu\nu}$ of a rotating black hole, where this metric possesses an event horizon and an ergo-region which cannot be denoted by a real-number valued field $\psi$ that is continuous in space-time. Far from the event horizon space is flat and the black hole can be approximated as a point, and so in this far-field limit $r >> l_p$ the point-like particles can be denoted by the non-physically-real term $\psi$ and the Kerr metric dropped from the expansion. In this limit that the gravitational effects of particles can be ignored, the Hamiltonian expansion is now exactly of the
form of the path integral expansion of quantum field theory [15], and the development of the quantum theory can proceed in the standard way from here.

6. Physics unification without quantum theory being fundamental?

This derivation of a quantum field theory only holds for the limit of flat space, so it cannot be combined with General Relativity to give a unified quantum theory. This origin for quantum theory implies there is no reason to abandon the classical physics of General Relativity as the incompleteness conditions do not arise for gravity, and consequently the unification of physics can only occur in classical physics through extending General Relativity. Such extra-dimensional Kaluza-Klein theories are known to be capable of unifying gravity with the particle forces such as electromagnetism [12], and they form the inspirational basis for higher dimensional quantum theories attempting physics unification. But this origin for quantum theory implies that the matter fields of such quantum theories cannot be fundamental. It also implies that the classification divide between particle and wave remains intact, and so there is no super-symmetry relating fermionic particles to bosonic waves. Such an elimination of the assumptions underlying unified quantum theories would just leave geometric extensions to General Relativity without additional fields. The problem with this is an apparent lack of particles.

Symmetry breaking is normally conceived of in terms of an added field, such as the Higgs field in the Standard Model, but is there another way? There a number of issues with a closed universe that are often overlooked in General Relativity, of which one is that a wormhole solution [16] is a hole, and inserting a hole into a sphere in any number of dimensions turns it into a torus, breaking the symmetry of the sphere. Standard considerations of Kaluza-Klein theories select the minimum number of dimensions able to accommodate the particle symmetries as being 11 [12], for which the closed universe is $S^{10}$ and a wormhole mediated symmetry breaking $S^{10} \rightarrow S^3 \times S^7$ gives a closed $S^3$ space and $S^7$ particle dimensions. The insertion of a hole can result a twisted torus given by the homotopy group relation $\pi_n(S^{n-1}) = Z_2 \forall n > 2$, where for $S^{10}$ this “twist” is of the form of a map from the particle $S^7$ to the $S^3$ spatial universe, giving a non-trivial vacuum structure. However, the sphere $S^7$ is a Hopf fibre-bundle [17] of an $S^3$ fibre over an $S^4$ base-space, and the non-trivial map is from the $S^4$ base-space to the $S^3$ universe, as $\pi_7(S^3) = \pi_4(S^3) = Z_2$. The map from $S^4$ to $S^3$ is given by selecting out a $S^1$ sub-space such that the remaining $S^3$ is then broken as it is mapped to the spatial $S^3$, and the net result is the sphere decomposition sequence:

$$S^{10} \rightarrow S^3 \times S^7 \rightarrow S^3 \times (S^3 \times S^4) \rightarrow S^3 \times (S^3 \times (S^3 \times S^1))$$

The symmetry group breaking for the $S^7$ particle space resulting from this sequence comes from the relation $SU(4)/SU(3) \cong S^7$ and is given by:

$$SU(4)/SU(3) \rightarrow (Spin(3) \otimes SU(2) \otimes U(1))/Z_3$$

where the $Z_3$ comes from the centre of SU(3). In this case, the unbroken U(1) symmetry gives the condition for the existence of topological monopoles [18], where there is a spectrum of
monopoles given by the different ways to apportion the $S^7$ dimensions to the $S^4$ base-space

$$\pi_7(S^4) = \mathbb{Z} \times \mathbb{Z}_{12} = \mathbb{Z} \times \mathbb{Z}_3 \times \mathbb{Z}_4.$$ 

This gives a 3 by 4 table of monopoles which have the same charges as the fundamental fermions, including $\frac{1}{2}$ electric charges for the monopoles with SO(3) colour charge. The details of the topology gives a spectrum of both electric and magnetic charged monopoles, where they also possess a $\frac{1}{2}$ topological spin charge. They are of the form of “knots” where space is wrapped around the compactified particle dimensions, and this gives particle-like black holes in the Kaluza-Klein theory where the interior of the $S^2$ sphere is devoid of space and so the event horizon is a real physical surface and there is no physical singularity.

This gives the necessary conditions for the incompleteness proof and derivation of quantum field theory given in the previous sections. The Kaluza-Klein theory is such that the non-trivial vacuum twist has the eigenvalues of the electroweak vacuum and the Hamiltonian is of the form of the Standard Model, but with additional terms involving the SU(2) (isospin) gauge fields and the scalar (Higgs) field. The geometry allows the derivation of closed formulae for the Weinberg angle, isospin coupling, hypercharge coupling and electric charge, and the Higgs coupling is $\lambda = \frac{\sqrt{2}}{2}$ predicting the Higgs boson mass to be exactly $\frac{1}{2}$ the electroweak scale. These closed formulae give values to within 1-2% of the experimental values of the Standard Model, despite them being derived solely within classical physics [15].

### 7. Foundational Questions

A simple extension of General Relativity to 11 dimensions is shown to provide a derivation of a quantum field theory for 3 families of 4 fermionic particles subject to 3 particle forces, with the force of gravity still being described by General Relativity [15]. Notably, the quantum field theory is derived from extended General Relativity and is of the mathematical form of the Standard Model of particle physics, except for the local colour symmetry group being SO(3) and not SU(3). The choice of colour group has been between these two, where SU(3) was chosen as quarks have $\frac{1}{3}$ electric charges, and because of quantum theory conditions. However, the $\frac{1}{3}$ electric charges arise here for colour group SO(3) [5] and it is natural to ask, do the quantum theory conditions still stand if quantum theory is not fundamental?

When the assumption of a fundamental quantum theory is dropped, the justification for introducing matter fields is lost and symmetry breaking fields and inflaton fields then look rather arbitrary and unphysical. But without matter fields, the only way to obtain particles in an extension to General Relativity is as topological defects in the structure of space, which requires a closed universe. This apparently conflicts with observation, but such observations are input into General Relativity with “constants”, which seems somewhat absolutist for a theory defined by the relativity of measurement. The classic example is the cosmological “constant” $\Lambda$, for which the natural relativity question to ask is, constant relative to what? The answer is the metric term next to it in the field equations, as is explicitly evident in the derivation of the field equations from the Einstein action using the variational principle. But the Friedman-Robertson-Walker metric of a closed $S^3$ universe is parametrised by the radius $a$ of the sphere, which is outside of the spatial universe in a notional 4th spatial dimension that doesn't really exist. The cosmological term $\Lambda(a)$ can be similarly parametrised by this extra-dimensional variable $a$ and still be “constant” with respect to variation in the metric $g_{\mu\nu}(a)$. In fact, it can be shown that a metric-field theory of a closed physical surface doesn't make physical sense unless the term $\Lambda(a)$
is parametrised in this way [5]. Concerns can also be raised about the gravitational “constant” $G$ and the “constant” speed of light $c$, asking the question: should $G(a)$ and $c(a)$ be parametrised by the radius of the universe? If so, cosmological observations are looking back at a time when the “constants” had different values, which could impact calculations sufficiently to affect the conclusion of whether the universe is open or closed.

The theory of section 6 might look as though it is going to be one of many possibilities, but it isn’t. For 3 spatial dimensions and the particle charges, it is the only purely geometric theory that gives those charges and is identified by the “simplicity principle” as it is the simplest possible unification of physics (for $\text{SO}(3)$ and $S^3$). More interestingly, it is characterised by the 4 spheres $S^0$, $S^1$, $S^3$, $S^7$ of: monopoles/anti-monopoles $S^0 = \{-1,+1\}$ in a $S^1$ cyclical closed $S^3$ universe with particle dimensions $S^7$. The interest lies with the relationship between metric spaces and normed division algebras, because there are only 4 normed division algebras over the real numbers – real numbers, complex numbers, quaternions and octonions – and the respective closed spaces in them are the 4 spheres $S^0$, $S^1$, $S^3$, $S^7$. The given $S^{10}$ scenario is the only way to obtain only these spaces in a purely geometric unified theory, and so will be uniquely identified by principles that specify only these spaces.

The real foundational question exposed by this derivation of quantum field theory [15] is, do there really exist observable features in classical physics that cannot be derived? There could be more at stake than the unification of physics because the non-derivable feature of the particle wave-property occurs in classical physics, and so other non-derivable features could also occur elsewhere in science, such as in biology, psychology, economics …
References

Technical Notes

Basic outline of conditions for incompleteness proof in classical physics [5]:

- Implication: if object state \( A \) is the cause of object state \( B \) and \( A \) exists, then state \( B \) will exist.
- Induction: if a statement about a set of objects is true for at least one object and is always true after one more object has been added to the set, then it is true for all set sizes.
- Initial functions are successor \( s(x) = x + 1 \), predecessor \( p(x) = x - 1 \), zero function \( z(x) = 0 \) \( \forall x \) and projection function \( P_i(x_1, \ldots, x_m) = x_i \) \( \forall x_1, \ldots, x_m \) giving the numbers of objects of each type.
- Addition: \( N \)-repeat control of the successor function \( s(x) \) gives the operation of addition \( x + N \) in the physically-real terms of object numbers changing through causal events.
- Multiplication: \( M \)-repeat control of the \( N \)-repeat control of addition above gives the operation of multiplication \( M \times N \) in physically-real terms.
- Substitution: A function \( f \) is obtained from functions \( g, h_1, \ldots, h_m \) by substitution when:
  \[
  f(x_1, \ldots, x_n) = g(h_1(x_1, \ldots, x_n), \ldots, h_m(x_1, \ldots, x_n))
  \]
  which will only be more than just a mathematical expression in theory when multiple object reactions form a tree-like cascade. E.g. object reactions \( A + B \rightarrow E \) and \( C + D \rightarrow F \) denoted by functions \( h_1 \) and \( h_2 \) over the number of reactant objects and give the number of product objects. If these react \( E + F \rightarrow G \) to give a new object type, the function \( g \) will be over the numbers of \( E \) and \( F \) objects produced by the initial reactions denoted by functions \( h_1 \) and \( h_2 \).
- Recursion: A function \( f \) is obtained from functions \( g \) and \( h \) by recursion when:
  \[
  f(x_1, \ldots, x_n, 0) = g(x_1, \ldots, x_n)
  \]
  \[
  f(x_1, \ldots, x_n, y + 1) = h(x_1, \ldots, x_n, y, f(x_1, \ldots, x_n, y))
  \]
  New variable \( y \) denotes the number of objects of a new type, which forms part of some pre-existing object transformation process already denoted within the theory.
- Unlimited recursion is required.
- Gödel number \( g \) can be constructed within the scope of the theory \( S(x) \) for any expression \( P \) in \( S(x) \) because \( S(x) \) contains arithmetic and every recursive number-theoretic function.
- The diagonal function \( D \) giving the Gödel number \( g \) of an expression \( P(g) \) taking as argument the Gödel number of itself can be constructed within the scope of \( S(x) \) by the above.
- Gödel and Rosser sentences can be constructed within the scope of \( S(x) \) by the above and the theory proven mathematically incomplete when \( S(x) \) is known to be consistent.

Vacuum eigenvalues

The dimensional reduction of maps from \( S^n \) to \( S^{n-1} \) \( \forall n > 2 \) can be given by the parametrisation \( x_{n-1} = x_\xi \cos \xi, x_n = x_\xi \sin \xi \) where the radius of the sphere \( r = x_0^2 + \ldots + x_{n-2}^2 + x_\xi^2 \) is reduced to that of \( S^{n-1} \). For the map \( S^4 \) to \( S^3 \) this parametrisation reduces the radius to that of \( S^3 \), where the \( x_3, x_4 \) coordinates define the \( S^1 \) group space of \( U(1) \) with group eigenvalue 1, and the coordinates \( x_0, x_1, x_2, x_\xi \) define the \( S^1 \) group space of \( SU(2) \) with group eigenvalue \( 1/2 \). As the \( S^1 \) fibre of the space \( S^7 \) doesn't participate in the map to the spatial \( S^3 \), the group eigenvalue is 0, selecting...
Spin(3), the double cover of SO(3), with group eigenvalues including 0. The non-trivial vacuum has the SO(3), SU(2), U(1) group eigenvalues (0, ½, 1), where the map corresponds to the configuration $U = i\hat{x} \cdot \sigma$ for which the gauge field is pure gauge, $A_\mu = U^{-1} \partial_\mu U$ as $U \in SU(2)$ [19], and the 2 configurations of homotopy group $\pi_4(S^3) = \mathbb{Z}_2$ can be given the chiral labels L and R.

**Weinberg angle**

Sphere $S^n$ is defined as the set $X$ of coordinate tuples $(x_0, \ldots, x_n)$ in $(n+1)$-dimensions for a given radius, and the parametrisation for the map of $S^n$ to $S^{n-1}$ gives the subset $Y \subset X$ of coordinate tuples $(x_0, \ldots, x_{n-2}, x_\xi)$ in $n$-dimensions. The subset $Y$ for $S^{n-1}$ contains the same coordinates $x_i$ for $i=0, \ldots, n-1$ with the same range $[-r_n, +r_n]$ and mean square over the set $X$ of $\langle x_i^2 \rangle = r_n^2/(n+1)$, and so the calculated radius $r_{n-1}$ of $S^{n-1}$ in terms of the $S^n$ coordinates is:

$$r_{n-1}^2 = \frac{n}{n+1} r_n^2$$

For the Electroweak vacuum of the map $S^4$ to $S^3$ where the group angle given by the eigenvalues is $\tan \phi_W = \frac{3}{2}$, the hypercharge radius $r_Y$ is the radius of $S^4$ and the isospin radius $r_I$ is the radius of the sphere $S^3$, the Weinberg angle $\theta_W$ will be given by:

$$\tan \theta_W = \frac{r_Y}{r_I} \tan \phi_W = \frac{1}{2} \sqrt{\frac{5}{4}}$$

This gives $\sin^2 \theta_W = 5/21 \approx 0.2381$ within the experimental range of $\sin^2 \theta_W \approx 0.23$ to 0.24 [20].

**Electric and magnetic fermion monopoles**

The SU(2) group space $S^3$ has two mappings to the spatial $S^2$ sphere depending upon the sphere decomposition selected: either $S^n$ to $S^{n-1}$ for which $\pi_3(S^2) = \mathbb{Z}_2$; or the fibre-bundle for which $\pi_3(S^2) = \mathbb{Z}$. The first gives topological monopoles with magnetic charges, whereas the second gives electric charges. The embedding of the broken SU(2) group space in the particle $S^7$ gives U(1) charge eigenvalues where the factor of 3 comes from the $\mathbb{Z}_3$ centre of SU(3):

$$\lambda_Q = \lambda_I + \frac{1}{2} \lambda_Y$$

where $\lambda_Y = -1$ for colourless monopoles and $\lambda_Y = \frac{1}{3}$ for coloured monopoles

For the corresponding mapping of the $S^3$ rotation group space to the spatial $S^2$ surface of the topological monopoles, the generic $S^n$ to $S^{n-1}$ map gives a $S^1$ rotation axis for which the rotation group eigenvalue is $\frac{3}{2}$, i.e. a fermion with spin $\frac{1}{2}$. 

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