Wigner called the effectiveness of Mathematics in Physics (and in natural sciences) "unreasonable". Against the widespread romantic position, I argue that what is unreasonable is the use of physical principles for founding physical theories. Physics without physics? This may seem an oxymoron. But the point made here is that the theory should be a purely mathematical construction, whereas its physical connotation should pertain only the interpretation of the mathematics. An exemplary case is that of group theory and physical symmetries. In contrast to the present call for mathematization, the current major physical theories either have mathematical axioms that lack physical interpretation, or have physical postulates. I therefore call for the construction of a theory that, though with limited (but relentlessly growing) domain of applicability, yet will have the eternal validity of mathematics. A theory on which natural sciences can firmly rely. This is what I consider should be the answer to the Hilbert’s call contained in his Sixth Problem.

Mathematics is eternal, Physics is temporary

Who does not believe that mathematics is eternal? Will the Pitagora’s theorem not continue to hold for the next millennia?

Differently from mathematical theorems, physical theories are not eternal. Should they be? At first glance such possibility may look overreaching: physics has always been riven with contradictions, and contradictions too often have played a pivotal role in the understanding of Reality. A Reality that, we believe, must itself possess internal logical coherence. But is transience an intrinsic limitation of physical theories, or is it just a temporary historical feature? We cannot deny that a similar phase occurred also for mathematics at its early pre-Hellenic stage, when the discipline was not so different from the “trial-and error” approach of physics (see e.g. the case of the value of Greek-Pi [1]).

It is commonly considered that a physical theory is in a sense eternal, since when replaced by a new theory it remains valid as a limiting theory within a narrower phenomenological domain. But is this really the case? Newtonian Mechanics is not a limiting theory of Quantum Mechanics: there is no procedure for recovering the classical from the quantum. On the other hand, Quantum Mechanics holds for the whole physical domain, but still relies on notions from the old classical theory, and the new theory is built
up over the relics of the old one through a “quantization” procedure that is not even well-defined.

The Sixth Hilbert Problem

Was impermanence of Physics the issue that motivated Hilbert to propose his Sixth Problem? The opening paragraph of the problem reads: “The investigations on the foundations of geometry suggest the problem: To treat in the same manner by means of axioms, those physical sciences in which mathematics plays an important part; in the first rank are the theory of probabilities and mechanics.” As Benjamin Yandell writes in his book about Hilbert’s problems [2]: “Axiomatizing the theory of probabilities was a realistic goal: Kolmogorov accomplished this in 1933. The word 'mechanics' without a qualifier, however, is a Trojan horse.”

What is ‘Mechanics’? Essentially anything that deals with forces and their effects on bodies. Which forces? Gravitational, electromagnetic, weak, strong, and … the last hypothesis of force. What do we mean by ‘body’? If we analyze at deep the notion of ‘physical object’, it evaporates into a mere “set of numerical coordinates”, as Quine says [3]. The notion of object is incompatible with something having properties and being itself object-composable, since there are properties of the whole that are incompatible with any possible property of the parts—a lesson that we learnt from Quantum Theory [4]. To make things worst, the notion of ‘particle’ as a localizable entity does not survive Quantum Field Theory [5]: what is left is only an irreducible representation of a group [6].

Group Theory as an exemplary case

It is not accidental that what endures time in theoretical physics is only its mathematical framework and the related mathematical notions and theorems. In this respect, as noticed by S. French [7], group theory provides a particularly interesting case study. It shows how physics can be formulated directly in mathematical terms, while being “physical” in its interpretation, exactly as it happens for Euclidean geometry. The main participants of group theory—primarily Weyl and Wigner, but also others such as Mackey and Wightman—initiated relevant advances on both physical and mathematical sides. And the success of group theory in the grand-unification of forces has been astonishing, starting from the creation of the notion of “isospin”, up to the quark model, while providing a geometrical categorization of the notion of elementary particle as a set of
invariances. It is also remarkable that in this way the notion of particle wears an ontological version of structural realism, claiming that all that is about the world is pure structure. The naive ontology is rationally replaced by mathematics, whereas the physical interpretation comes in terms of relational structure of reality, a la Carnap [8]. Thus the mathematization of physics provides a unique reconciliation point between realist and antirealist views.

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Failure of previous mathematization programs

The route for the mathematization of physics has been deeply explored by Wightman [9] for Quantum Field Theory, however, with the physics still hinging on mechanical definitions from the old classical field theory, including a background space-time. Moreover, the mathematical framework is inherited from Quantum Theory, based on its Hilbert-space axioms that have no physical interpretation. Irreconcilable with Quantum Field Theory, General Relativity—the other main theory in physics—is based on an “equivalence principle” which is physical, whereas the dynamical equation (Einstein’s) are heuristically derived. No wonder that, along with the successes of both general relativity and of algebraic field theory (CPT and spin-statistics theorems), in the presence of non-mathematical principles and in the lack of physical interpretation of the mathematical axioms, the two theories are doomed to remain at clash and to prove internal logical incoherence (see e.g. the Haag’s theorem1).

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The mathematization of Physics

The reader will now ask: how is it possible to axiomatize a physical theory mathematically, without resorting to physical notions? Physics without physics? It is not an oxymoron? The answer is to keep physics only at the interpretational level for both axioms and theorems. This means that physics emerges from the mathematics through the ability of portraying phenomena and of connecting experimental observations. A magnificent example is the case already mentioned of SU(3) symmetry that lead to the quark model, which succeeded in explaining a very extensive phenomenology in particle physics. The reader may complain that the same SU(3) symmetry lacks itself a physical interpretation: my answer is that this is an hallmark of the lack of physical interpretation of the Hilbert-

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1 Haag’s theorem states that there can be no interaction picture—that we cannot use the Fock space of noninteracting particles as a Hilbert space—in the sense that we would identify Hilbert spaces via field polynomials acting on a vacuum at a certain time. The Constructive Quantum Field Theory program has shown that Quantum Field Theory is only an effective theory.
space quantum axioms, which must be substituted with meaningful principles. The SU(3) symmetry is only the last of a list of quantum features: the isospin and the spin itself physically come out from the mathematics of group theory. In such respect the Dirac equation is the archetype of a mathematical theorem with physical interpretation, with the physics of spin and antimatter emerging from it.

It must be remarked that what makes complete and coherent the physical interpretation of the mathematically formulated theory is having also all axioms endowed with physical interpretation. We already noticed that this is not the case of the Hilbert-space axiomatization of Quantum Theory. In order to be purely mathematical, the axiom should not even contain words requiring physical definition, although being appropriate in describing a physical scenario. What such a scenario would be? It can be an epistemological context. This is the case of the Quantum-Logic program of von Neumann [10], who after having mathematically formulated Quantum Theory [11] sought its interpretation as a new kind of logic. Indeed, as it was shown by Jaynes [12] and Cox [13], probability theory is an extension of logic, and quantum theory is a special kind of probability theory. What has been missing in the quantum-logic program, however, is the connectivity among events, which is the special trait of the operational axiomatizations of quantum theory [14,15], and mathematically is the monoidal category theory [16].

The immense power of having both axioms and theorems endowed with physical interpretation is the availability of deductions that are physically meaningful at each logical step. This kind of reasoning builds up a logically grounded physical insight, compared to a mathematical derivation with no physical reading following from axioms that lack physical interpretation. Again, this is the case of the Hilbert-space-formulated Quantum Theory, which for almost a century rested on mathematical postulates that still engender controversial interpretation. No wonder that the same mathematical formalism of the theory too often lacks physical interpretation, with the result that commonly held principles as “locality” and “causality” are formulated in a way that they are seemingly violated by the theory.

Axioms with physical interpretation: informationalism and structuralism

Ultimately, the only motivation for introducing physical notions in the axioms of a theory is the wish of connecting it to physical ontologies. In a structuralist point of view what matters are only the relations among the ontologies, the theory being only an

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2 Putnam’s internal realism [17] is also very close to the structuralism of Carnap [8].
omomorphism of reality. This is exactly the spirit of an algorithmic description of reality, where the latter emerges from a network of mathematical relations. An authentic paradigm of such approach is “reality as a simulation”. In other words: “reality as pure software”, or “software without hardware”. This is the informational paradigm for physics. An example are the six informational axioms of quantum theory [14], mathematically defined within the framework of monoidal category theory, and having physical interpretation in terms of experimental protocols, namely “physical algorithms”. Ultimately all the axioms of the theory are requirements for the falsifiability of theoretical propositions in conditions of experimental control. Quantum theory thus becomes a theory of “processes”, i.e. relations without noumena, in describing the network of connections between events, some of which are under our control (the “preparations” or “states”), some other are what we observe (the “observations” or “effects”), some other are theoretical “transformations” in between preparations and observations. The field theoretical description corresponds to adding rules to the process in terms of general topological requirements for the connectivity network (e.g. its homogeneity, locality, and isotropy), corresponding to minimizing the algorithmic complexity of the process. From such rules the “mechanics”—including space-time and Lorentz covariance—emerges [18,19] at large scales of a connectivity network that is inherently discrete. Figuratively, physics emerges in form of computational patterns over an immense quantum-digital screen.

Method

Once the axioms are stated, the advancement of the physical theory proceeds by devising and thereafter proving statements of physically meaningful theorems, and in parallel, by evaluating physically meaningful results for particular instances of theorems and/or special values of variables. The first aim is to build up an increasingly detailed and faithful description of observations, and correspondingly deepening the physical interpretation of the mathematics, ultimately achieving a vision of the world and a logical understanding of the observations. The second aim is to design experiments apt to falsify the theory, namely to refute the physical interpretation and/or the suitability of some of the axioms of the theory. Infinity—whether in the large or in the small (i.e. the continuum)—is metaphysical, and should be taken for mathematical convenience, as a limit for evaluating leading orders in asymptotic behaviors. More generally, a constructive mathematical framework should be take, e.g. avoiding the axiom of choice: physics can

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3 It should be noted that the same can be said of classical theory, which indeed share five postulates with quantum theory. The distinctive axiom of quantum theory is the so called “purification axiom”, that provides a control of randomness [14].
emerge only from the solution of problems that are decidable and/or effectively solvable. Theoretical connections should be confined within a finite causal cone for a finite time interval.

We are now left with the main problem of how to choose the axioms of the theory. First, we need to establish axioms needed to guarantee the same scientific nature of the theory. This must include the logical framework of the theory, along with the requirements for its falsifiability under experimental control: these must be independent on the physical content of the theory. We have seen that the axioms for quantum theory of Ref. [14] are exactly of this kind. After this stage, theoretical convenience dictates considering axioms that minimize the algorithmic complexity of the theory—e.g. symmetries, invariances, linearity, minimization of dimensions of linear spaces, etc. The resulting set made with these starting axioms will constitute a backbone theory, to which one can add more specific mathematical axioms having physical interpretation coming from heuristics, idealizations, analogies, etc., always following the general rule of minimizing the algorithmic complexity of the theory. The more universal the axioms are, the more stable and general the theory will be.

The “beauty of the mathematics” of the theoretical description should not be taken as a primary criterion. Beauty may depend on the particular theorem of the theory, on the specific mathematical representation, and on the physical scale of application. A theory may look ugly at some scale or for some representation, and may appear beautiful at a different scale or for a different representation. For example, a discrete quantum automaton theory [18] can give rise to emergent Lorentz transformations that become awful nonlinear at very high momenta [20], however, the assumptions of such a theory are minimal compared to those of quantum field theory, and in addition the theory is constructively mathematically well defined, e.g. with no ultraviolet divergencies. Beauty is in the eye of the beholder!

Finally, it is obvious that in a mathematically formulated theory all variables with physical interpretation will necessarily be adimensional. Therefore, the physical interpretation of the theory will be complete only if the theory itself contains a way to establish the units of measurements of the variables of the theory. This can be done only by referring to special values (e.g. maximal or minimal allowed values) of the adimensional variables of the theory that are pregnant with physical interpretation. An exemplary case is again that of the quantum cellular automata theory of fields [18], where unitarity leads to an upper unit-valued bound for the particle mass, and the framework implies a minimal distance and a minimum time. If the maximum mass value is interpreted as the Planck mass (which follows by either a mini-black-hole interpretation, or by assuming that Newton gravity holds as an order of magnitude at the discrete scale), then from the free quantum
field theory emerging at large scales one finds out that the units are the Planck units for length, time, and mass.

Conclusions

Is mathematization of Physics premature? Although we may never achieve a complete set of axioms for physics, yet we can have stable subsets of axioms (along with alternative equivalent sets), and derive a stable collection of theorems, whose physical interpretation will provide us with physical truth. “Truth” by definition must be evident and eternal, as for mathematical truth. Mathematics is an evolution of the human language, the ultimate idealization and synthesis of observations grouped into equivalence classes. Physics is something beyond logic, and because of this can be only taken as an interpretation of Mathematics. A thorough mathematical axiomatization program for physics will open a new era, less speculative and more logical, more focused on seeking general principles for physics, whereas all results from the principles will retain autonomous mathematical value. Four hundred years already passed after Galileo introduced the mathematical description of physics: the mathematization of physics will initiate the next, maybe the ultimate, Galilean revolution.

The world is mathematical [21]. The world is truly mathematical.

Disclaimer

The fact that the author calls for a mathematization of Physics does not imply that he is a good mathematician.
Reference to author’s pet theories is only for the sake of exemplification, although these theories were designed and elaborated exactly within the spirit and motivations of the present essay. The reader should judge the present mathematization program independently on his opinions on these yet-too-young theories.
The reader who considers the proposal of mathematization of Physics preposterously unfeasible has already given up the possibility of acquiring true knowledge in science.
References


