Simple Math for Questions to Physicists

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Abstract

This is an example of using simple mathematics to ask physicists (hopefully) interesting questions. A nonstandard model of time illustrates the approach.

1 Introduction

Although an answer to a question might be expressed in one language, the question it answers could be expressed in a different language. The process still works. Someone could ask a question in Spanish and get an answer in English. Whatever the language of the questioner, the answer still exists.

This essay presents questions, not answers. The language for expressing the questions will be English and simple uses of situation theory, channel theory, nonstandard analysis, non-wellfounded sets, and game theory.

2 A nonstandard model of time

Along with the standard model of time there is a nonstandard (Robinson 1996) model of time.

Here’s one way to write the standard model of proper time:

\[ \text{time} = (\text{now}, \text{time}) \] (1)

It’s a "stream" (Barwise and Moss 1996), shorthand for:

\[ \text{time} = (\text{now}, (\text{now}, (\text{now}, (\text{now}...)))) \] (2)

Equation 2 signifies a re-writing process starting with the occurrence of "time" on the right hand side of the equal sign in equation 1. "Time" on the right hand side is re-written "(now, time)," over and over again, signifying the stream of proper time and leading to equation 2.

Each re-writing of "now" in equation 2 re-maps the natural language symbol "now" to a real number which is the index of a point on the curve of proper time.

Next a nonstandard model. Instead of re-mapping "now" to a real number, by invoking nonstandard analysis "now" can be re-mapped to a nonstandard "monad."

In a nonstandard model, around each standard real number in "now" there will be a "halo" of nonstandard real numbers infinitely close to the standard part of the monad.

The standard part of the monad now is a real number. In front of and behind the standard part of each monad there is a halo of nonstandard real numbers. The halo before the standard part of this type of monad is called here "the nonstandard future." The halo behind is called "the nonstandard past."

\[ \text{now} = (\text{nonstandardFuture}, \text{standardNow}, \text{nonstandardPast}) \] (3)
3 More language

This introduces some mathematical languages, of which I will use a few words in the balance of the essay.

3.1 Non-Wellfounded Sets

Obviously Equations 1 and 2 also involve another mathematical language. It’s the language of Non-Wellfounded Sets (Barwise and Moss 1996), applied to model streams. Those familiar with recursion in computer languages will be somewhat familiar with the application. The model of time comes from Barwise and Moss (1996, 3).

3.2 Situation Theory

Criticism of nonstandard analysis focuses on the difficulty of model theory. Situation Theory (Barwise 1988) might be a less time-consuming approach for a physicist wanting to understand how languages and models work together. In Situation Theory the model becomes a situation. A situation differs from a model in that a situation can be a constituent of itself.

That, of course, is circular, which explains the above work on non-wellfounded sets by the author of The Situation in Logic, Jon Barwise. Non-wellfounded sets deal with circularity and so they’re useful for modeling situations.

3.3 Channel Theory

Information Flow: The Logic of Distributed Systems is about Channel Theory. The "infomorphism" becomes important. More on that below.

3.4 Game Theory

Rules, moves, winning, losing are informal concepts which are made mathematically formal in Game Theory.

4 A Nonstandard Model of Existence

Having considered "time," then "space" might seem a good next step. But on ScienceFriday (2/27/15) – when asked about a concept in physics that he would abandon as being basic– Sean Carroll answered "space." The next section inquires, not about space, but information and possibilities.

4.1 A good model of information and possibilities

In Barwise (1997), there is a model of information and possibilities first expressed informally and then mathematically using Channel Theory. For reasons that will be apparent later, here we want to follow that outline but use numbers instead of the local logics of Channel Theory.

"The main idea of informationalism is to take the inverse relationship between information and possibility as a guiding tenet. The Inverse Relationship Principle: Whenever there is an increase in available information there is a corresponding decrease in possibilities, and vice versa. The implications of this principle will depend a lot on what else one assumes about information and possibilities. My main proposal here is that a good theory of possibility and information should be consistent with this principle. As I analyze things, impossibilities are those states of the system under investigation that are ruled out by (i.e., incompatible with) the currently available information about the system. States not so ruled out are possibilities." (Barwise 1997)

Here’s the informal model from Barwise (1997):
issues the set of all states of a given inquiry depends on the system under investigation and on the issues regarding the system relevant to the inquiry.

States A state is a way of resolving all the relevant issues.

impossibilities The set of possible states at a given point in the inquiry depends on the information concerning the issues currently available. The impossible states are those incompatible with the currently available information; the others are possible.

Available information What information is available at any point in an inquiry is a context-sensitive matter, depending on the kind of possibility one is considering and on the progress of the inquiry up to that point.

Increases in information The correct elimination of any nonempty set of possibilities corresponds to a strict increase in the information available at the next stage in the investigation.

Decreases in possibilities Conversely, the acquisition of any new information corresponds to a strict decrease in the states that are possible." (Barwise 1997)

The goal of the following sections is to find a “good theory of possibility and information” by using numbers to represent states, instead of the local logics in Channel Theory.

"...our choice of states is determined in part by the issues raised by the rules of the game." (Barwise 1997)

4.2 The issue of existence

Given a nonstandard model of proper time, the first issue is existence. Equations 1-3 comprise the field of play for what might be called "The Existence Game for a Particle."

Rule 1: When a particle exists, it is not a field. What makes a particle a particle is that it does not occupy all of the basic states at the same time. A particle occupies only one basic state at a time.

And at that time, it does not occupy any other basic states in the game.

Here we notice that nothing so far eliminates a state which is a system of basic states. This will be important for considering information in the nonstandardFuture.

Following Rule 1, the possibilities for a particle work like this: A particle existing in the standardNow means that in the nonstandardFuture there must be a logical "Anding" or product of two different kinds of possible states. One of the states in this product is a basic state, where the particle can exist in only one such state. Call that state X.

The other state in this product of two states is a state which is a system, specifically a system comprising those basic states in which the particle cannot exist because it exists in X. Call this state Y. Y is like a mirror of X, where the image is reversed.

The product of these two different kinds of states is XY. In the nonstandardFuture, only the product XY can carry information about a particle that exists in the standardNow. For in the nonstandardFuture the particle is unknown, and only states are known. Only by means of the product XY in the nonstandardFuture is information available about a particle, since particles are unknown there.

To resolve the issues of Rule 1, a type of number is needed that works as above. That is, to convey information about the particle, a type of number representing the basic states must be able to form a product with two factors where one number of this type represents one, basic state and another number of this type represents a system of all the other basic states, in effect a mirror with a reversed image of the first number.

The product holding means the possibility that one basic state is occupied and also the possibility that no other basic state is occupied. In this way information about possible states and systems of possible states can carry information about the particle, even though the particle doesn’t exist in the nonstandardFuture.
4.3 The same state can be possible or impossible

This requirement of the above model can be satisfied by picking a type of number for a state that can be set to zero, which means the state is impossible. A nonzero number means the state is possible.

4.4 Possibilities are logical

That possibilities are logical is basic in Barwise (1997). The mathematical model there is built on the local logics of Channel theory. So for the type of number needed to model state it must not matter which state comes first or second in a multiplication or addition operation. The desired type of number for representing states must be commutative.

Also because possibilities are logical, it should be a type of number that cannot be compared to another by the greater-than/less-than ordering relation.

4.5 Increasing or decreasing possibilities

For a good model, the type of number we need for representing state must support The Inverse Relationship Principle.

For a number, multiplication and addition are ways of increasing possibilities by mathematically composing the numbers. Decreasing the number of possibilities involved then means the type of number desired must support subtraction and division. Division is how a possibility can be factored away from a product of possibilities. So the type of number needed must support division.

This alone is very close to completely specifying, first, one of the four normed division algebras (real, complex, quaternion, or octonion). With the above requirements of commutativity and non-ordering, the complex numbers stand alone. But the norm is also relevant (see below).

4.6 The issue of information

Another issue in the existence game is that, in each of the monads now, all three situations, nonstandardFuture, standardNow, and nonstandardPast must support information about the same particle. This is where the infomorphism of Channel Theory will be required.

4.6.1 Modeling information in the nonstandardFuture and nonstandardPast

In Situation Theory an element of information, termed an "infon," is written as

\[ s \models \sigma \]  \hspace{1cm} (4)

\( s \) is a situation. \( \sigma \) is the type of the situation. In this model, information does not exist in a situation by itself, when the situation is untyped. If \( s \) is of no type at all, then in this model no information exists. If \( s \) with no type at all were a part of a system, in this model there is no way that any other part of the system could carry information about \( s \). In fact, nothing else anywhere could carry information about \( s \). For other parts of the system to carry information about \( s \), it must be of some type, for example \( \sigma \). More about this below.

Using Situation Theory, an element of information carried in the nonstandardFuture can be written

\[ \text{nonstandardFuture} \models XY \]  \hspace{1cm} (5)

Where \( X \) is the number associated with the possible state \( x \), and \( Y \) is the number associated with the possible state \( y \). \( x \) is one of the possible states for the particle, and \( y \) is the state of Not being in any state other than \( x \). The symbols will be changed later when the choice of type of number to resolve all the issues is concluded.

An element of information carried by the nonstandardPast can be written

\[ \text{nonstandardPast} \models Z \]  \hspace{1cm} (6)
4.6.2 The informorphism

To make the information signified by equation 5 the same as the information signified by equation 6, we need an "infomorphism."

Barwise worked on informorphisms since before 1988, when his book *The Situation in Logic* was published.

"The final analogy I want to draw from topology comes from the proven importance of studying not just topological spaces themselves, but relationships between different spaces. Indeed, it has turned out that one of the most important notions of topology has been that of a homeomorphism, a function from one space to another that respects nearness relations in an appropriate way. I think that something similar is going to happen here. I suspect that we are going to find ourselves studying 'infomorphisms,' maps from one information space to another that preserve information. But before we can define these maps, we need to understand the basic structure they need to preserve." (Barwise 1988, 256)

Eight years later the book *Information Flow*, was published. There we see the definition of the infomorphism. There is a biconditional on page 32 called the *fundamental property of infomorphisms*. We transfer this biconditional into the nonstandard monad *now* in equation 3:

\[
\begin{align*}
\mathcal{f}^{\land}(&\mathcal{nonstdPastPt}) \models_{\mathcal{nonstdFuture}} \mathcal{XY} \\
\text{iff} \\
\mathcal{nonstdPastPt} \models_{\mathcal{nonstdPast}} \mathcal{f}^{\land}(\mathcal{XY})
\end{align*}
\] (7)
In the context of equations 1-3, the informorphism looks like a loop. But on each iteration the numbers representing possibilities can change.

To resolve the issues of preserving information about the particle within the now monad of non-standard time (equation 3) a type of number must be chosen for possible states which establishes an informorphism between the nonstandardFuture and the nonstandardPast.

Ψ in the following equation works:

\[ ΨΨ^* = P \]  \hspace{1cm} (8)

It’s the Born rule. So it seems appropriate to call this informorphism "The Born Infomorphism.”

And now it’s clear that the normed division algebra of the complex numbers can be used to model possible and impossible states, and satisfy the Inverse Relationship Principle.

Note: To make this work the Heisenberg picture must hold within the monad now of nonstandard time. The wave function or complex number has to remain constant during now for equation 7 to
5 More inquiry into the mathematical game

Seeing a mathematical problem as a game seems to be a noncontroversial way to find theorems (e.g. Shafer and Vovk 2014). Heeding Sean Carroll’s comment, above: Is seeing a mathematical problem as a game an approach useful for new explorations, prior to introducing space into models?

There is perhaps a suggestive analogy to action in physics. A dual acting spring with a weight, extended to some point, released, then stopped again when returning to that location, records zero action (see for example, Physics Stack Exchange 2014). In a game, the player is given time to make a move. "Moving" to the same spot also results, in some sense, in zero action—moving to the same spot is the same as not moving at all. Question: would this provide a mathematical approach to some kind of action, perhaps as above, before space is introduced?

The principle of least action involves an extremum. There is in fact an extremum for the game of Probability Learning (Gallistel 1990, 351), which is solved by the player balancing two kinds of conflicting regret (see below). By analogy the "action,” or the moves in the probability learning game, balance conflicting player regrets about moves— or about "action."

Here is the probability learning experiment:

A subject animal is given the opportunity to move to one, of more than one, possible food supplies. The subject does not know which possibility will provide food. For each move in the game, only one of the possibilities will provide food. The subject sees when the food occurs at some possibility other than the one chosen. The game continues for a number of moves.

Here is the finding: the subject will move to each possibility with the same probability as the experimenter had determined beforehand for food occurring at each possibility. The subject learns the probability for each possibility as determined beforehand by the experimenter.

Say for each possibility the experimenter has set up a probability \( e_i \). And the subject moves to that possibility with probability \( s_i \).

From a history of moving to possibility \( i \), but seeing the food occur elsewhere, the subject experiences regret in amount \(-\ln(s_i)(1 - e_i)\).

From a history of moving to some other possibility than \( i \), but seeing the food occur at \( i \), the subject experiences regret in amount \(-\ln((1 - s_i)(e_i))\)

These are two opposing forces of regret. They balance when

\[-\ln((s_i)(1 - e_i)) = -\ln((1 - s_i)(e_i))\] (9)

The solution is probability learning:

\[s_i = e_i\] (10)

Equation 10 looks somewhat like equation 8.

It seems possible to think of the Existence Game for Particles as a game of probability learning. The nonstandardFuture would play the role of the experimenter in the probability learning game. And the nonstandardPast would play the role of the subject in the probability learning game.

6 Final questions

Having performed an informationalist inquiry by using simple mathematics and considering a game, finding the rules, considering the issues inherent in those rules, the ways of resolving those issues in terms of states, and information involved, we find the Born infomorphism. All of which, now becomes the context for a few final questions.

Equation 4 supports the idea that something which exists (a situation) may or may not support information as to what type it is, or what state it might be in– because it may be "pre-state." For
those familiar with computer languages, there is something familiar in this idea. “Both the object-based approach and the stream-processing approach raise significant linguistic issues in programming.” (Abelson and Sussman 1996, 218) There is object-oriented programming, and there is stream or function based programming. The difference is that objects have state. But the functions used in stream-oriented programming do not have state.

In the informationalist inquiry attempted here, states are ways to resolve the issues raised by the rules of the game. But is physical reality more like programming than we suspect? Can a pure process—like a situation that can’t be typed or occupy states, or like a stream-based computer which presents no objects with state to the programmer—actually exist in the physical world? And since it does seem to have some kind of “pre-space” action to it, is the Born infomorphism like a situation with no states, or a stream-based function with no states? It does cycle, in terms of equations 2 and 7—which may seem analogous to certain kinds of action in physics.

However, if there is some kind of action associated with the Born infomorphism, would the ”Lagrangian” involved always be negative. Why? Because compared to the case where potential energy depends on what state in which we find the object, in the case of a pure process or function processing streams, there are no states that it occupies. Would this mean it has, in that sense, no potential energy for inclusion in some kind of Lagrangian because potential energy depends on being in states? And would this be a kind of Lagrangian that is always negative?

The cycle spinning around in figure 1 is not necessarily a cycle that returns to the same spot every time. Because for each loop the product $\Psi \Psi^*$ may change. Interactions would have to change that product.

So figure 1 is perhaps a picture of two kinds of action. First, the action associated with changes in the product $\Psi \Psi^*$. Secondly, the cycling of the Born Infomorphism itself.

Is the Born Infomorphism a stateless process, with negative Lagrangian, that’s required to enable the existence of Lagrangians for the particles so served?

From the Chandra X-Ray Observatory web site (NASA 2006):

”Dark matter and normal matter have been wrenched apart by the tremendous collision of two large clusters of galaxies.”

If the Lagrangians and derived action of dark matter and normal matter are of different types, could it also be that the two behave differently with regard to $E = mc^2$? So when there are big collisions like this, normal matter is converted to light. Leaving the dark matter behind, and perhaps leaving a new kind of galaxy, comprised of nothing but spinning Born infomorphisms, with every $\Psi$ in every infomorphism vanishing to zero.

Final question: Is ”action” more basic than space?
References

Abelson, Harold and Gerald Sussman with Julie Sussman 1996. *Structure and Interpretation of


