Quantum correlations with indefinite causal order

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FQXi Conference, Vieques, January 7th
Causality & space time structure

$A \prec B$: A before B
$A \sim C$: A and C causally neutral
Quantum causal relations?

Space-time distance between A and B is not well-defined.
Outline

- Framework for quantum mechanics with no assumed global causal structure:
  
  includes all causally ordered (spatial and temporal) situations: shared states, channels, channels with memory, and probabilistic mixtures of these.

- Correlations that defy causal order:

  Violation of a “causal inequality” – a communication task that cannot be accomplished with causally ordered operations.

Causal order from space-time

**No-signalling**

- Space-like separated
- Causally neutral

**One-directional signalling**

- Alice before Bob

Time-like separated
Operational meaning of signalling

No-signalling
Space-like separated

\[
\sum_a p(a, b | x, y) = p(b | y)
\]
\[
\sum_b p(a, b | x, y) = p(a | x)
\]

One-directional signalling
Time-like separated

\[
\sum_a p(a, b | x, y) = p(b | y)
\]
\[
\sum_b p(a, b | x, y) = p(a | x, y)
\]
Two-directional signalling?

Causal Loop

Gödel Universe: **Closed time-like curves** (CTC)

Proposed quantum solutions: Deutsch’s or the Bennett-Schumacher-Svetlichny-Lloyd CTC-like structures are **non-linear** extensions of quantum theory

Linear structures, free of paradoxes?
The framework: Closed laboratory

A system enters the lab. A transformation is performed, and an outcome $j$ is obtained. The system exits the lab.

This is the only way how each party interacts with the “outside world”.

No prior assumption of pre-existing causal structure, in particular of the pre-existing background time.
Main premise:

Local descriptions agree with quantum mechanics

Transformations = completely positive (CP) trace-nonincreasing maps

\[ \mathcal{M}_j : \mathcal{L}(\mathcal{H}^1) \rightarrow \mathcal{L}(\mathcal{H}^2) \]

\[ \mathcal{M} \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2) \]

\[ \mathcal{M} \downarrow \text{CPTP} = \sum_j \mathcal{M}_j \]

Completely positive trace preserving (CPTP) map
Two (or more) parties

Local CP maps

Goal: characterize the most general W

Probabilities are bilinear functions of the CP maps

\[ P(\mathcal{M}^A, \mathcal{M}^B) = \text{Tr} \left[ W^{A_1A_2B_1B_2} \left( M^{A_1A_2} \otimes M^{B_1B_2} \right) \right] \]
Bipartite probabilities

1. Nonnegative probabilities:
   (ancillary entangled states do not fix causal order)

   \[ W^{A_1A_2B_1B_2} \geq 0 \]

2. Probability is 1 for all CPTP maps.

   \[ \text{Tr}\left[ W^{A_1A_2B_1B_2} \left( M_{CPTP}^{A_1A_2} \otimes M_{CPTP}^{B_1B_2} \right) \right] = 1, \]
Terms appearing in process matrix

\[ W_{A_1 A_2 B_1 B_2} = \sum_{\mu_1, \ldots, \mu_4} a_{\mu_1 \ldots \mu_4} \sigma_{\mu_1}^{A_1} \otimes \ldots \otimes \sigma_{\mu_4}^{B_2} \]

\begin{align*}
\sigma_i^{A_1} \otimes 1_{\text{rest}} & \quad \text{type } A_1 \\
\sigma_i^{A_1} \otimes \sigma_j^{A_2} \otimes 1_{\text{rest}} & \quad \text{type } A_1 A_2
\end{align*}

\[ \ldots \]

<table>
<thead>
<tr>
<th>States</th>
<th>Channels</th>
<th>Channels with memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1, B_1, A_1 B_1 )</td>
<td>( A_2 B_1 )</td>
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Causal order between parties

\[ W^B \not\leq A \quad \text{Bob cannot signal to Alice} \]

\[ W^A \not\leq B \quad \text{Alice cannot signal to Bob} \]

Probabilistic mixtures of ordered processes:

\[ W^{A_1 A_2 B_1 B_2} = q W^B \not\leq A + (1 - q) W^A \not\leq B \]

Are all \( W \) of that form? \[ \text{No!} \]
Causal game: Guess partner‘s input

- Alice is given bit \( a \) and Bob bit \( b \).
- Alice produces \( x \) and Bob \( y \), which are their best guesses for the value of the bit given to the other.
- Bob is given an additional bit \( b' \) that tells him whether he should guess her bit (\( b'=1 \)) or she should guess his bit (\( b'=0 \)).
- The goal is to maximize the probability for correct guess:

\[
p_{\text{succ}} := \frac{1}{2} \left[ P(x = b|b' = 0) + P(y = a|b' = 1) \right]
\]
Causally ordered situation

Case: $B \preceq A$

Global Time

$P(y = a | b' = 1) = 1$

$P(x = b | b' = 0) = 1/2$

$p_{succ} := \frac{1}{2} [P(x = b | b' = 0) + P(y = a | b' = 1)] \leq \frac{3}{4}$
Indefinite Causal Structures

\[
W^{A_1A_2B_1B_2} = \frac{1}{4} \left[ \mathds{1}^{A_1A_2B_1B_2} + \frac{1}{\sqrt{2}} \left( \sigma^A_{z} \sigma^B_{z} + \sigma^B_{z} \sigma^B_{z} \right) \right]
\]

two-level systems

\(b'=1\): Bob measures \(\sigma^B_{z}\)
Indefinite Causal Structures

\[ W^{A_1A_2B_1B_2} = \frac{1}{4} \left[ 1_{A_1A_2B_1B_2} + \frac{1}{\sqrt{2}} \left( \begin{array}{c} A_2 \\ B_1 \\ z \end{array} \right) + \sigma_z^{A_1} \sigma_x^{B_1} \sigma_z^{B_2} \right] \]

\[ b' = 1: \] Bob measures \( \sigma_z^{B_1} \)

\[ b' = 0: \] Bob measures \( \sigma_x^{B_1} \)
Indefinite Causal Structures

\[
W^{A_1A_2B_1B_2} = \frac{1}{4} \left[ \mathbb{1}^{A_1A_2B_1B_2} + \frac{1}{\sqrt{2}} \left( \sigma_z^{A_2} \sigma_z^{B_1} + \sigma_z^{A_1} \sigma_x^{B_1} \sigma_z^{B_2} \right) \right]
\]

Depending on his choice Bob can end up after or before Alice with probability \( \sqrt{2}/2 \)

The probability of success is: \( p_{\text{succ}} = \frac{2 + \sqrt{2}}{4} > \frac{3}{4} \)
Progress

\[ \frac{2 + \sqrt{2}}{4} \] is the Tsirleson’s bound for quantum correlations with indefinite causal order

• One has causally non-separable process matrices that do not violate causal inequality (analogous to the relation between nonlocality and entanglement)

• „GHZ-type“ of correlations for three observers [arxiv: 1312.5916 Ämin Baumeier, Stefan Wolf]
Summary

• Unified framework for signalling (time-like) and no-signalling (space-like) quantum correlations.

• Situations where the causal order between laboratory operations is not definite → global causal order need not be a necessary element of quantum theory.
Outlook

• Can we realize non-causal processes in the lab?

• A generalization of concept of space-time?

• Principles that select the generally signalling correlations allowed by QM?

• A new resource for quantum information processing?

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Thank you for your attention

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Terms **not** appearing in process matrix

\[ W^{A_1 A_2 B_1 B_2} = \sum_{\mu_1, \ldots, \mu_4} a_{\mu_1 \ldots \mu_4} \sigma_{\mu_1}^{A_1} \otimes \ldots \otimes \sigma_{\mu_4}^{B_2} \]

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Where to look for non-causal processes?

1. Within standard quantum mechanics? It may be possible to create causally non-separable situations making clever use of superposition and entanglement.

Pavia group, Chriribella et. al
Conclusions

• **[Not shown]**: In the classical limit all correlations are causally ordered

• Unified framework for both signalling (“time-like”) and non-signalling (“space-like”) quantum correlations

• Situations where a causal ordering between laboratory operations is not definite → Suggests that causal ordering might not be a necessary element of quantum theory
Choi-Jamiołkowski isomorphism

CP maps

\[ M : \mathcal{L}(\mathcal{H}^1) \rightarrow \mathcal{L}(\mathcal{H}^2) \]

Positive matrices

\[ M \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2) \]

Maximally entangled state

\[ |\Phi^+\rangle = \sum_i |i\rangle |i\rangle \]

\[ |i\rangle \in \mathcal{H}^1 \]
Outlook

• Can we realize non-causal processes in the lab?

• A generalization of concept of space-time?

• Principles that select the generally signalling correlations allowed by QM?

• Is $\frac{2 + \sqrt{2}}{4}$ a “Tsirelson bound for non-causal correlations”?

• A new resource for quantum information processing?

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Thank you for your attention
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Bipartite probabilities

Bilinear functions of the CP maps:

Representaion

\[
P(\mathcal{M}^A, \mathcal{M}^B) = \text{Tr}
\left[
W^{A_1 A_2 B_1 B_2} \left( M^{A_1 A_2} \otimes M^{B_1 B_2} \right)
\right]
\]

„Process matrix“

Choi-Jamilkowski representation of CP maps

Goal: characterize the most general W
Where to look for non-causal processes?

1. Within standard quantum mechanics? It may be possible to create causally non-separable situations making clever use of superposition and entanglement.

Pavia group, Chriribella et. al
“In summer with a large amount of ice-cream consumption there are lot of sun-burn cases.”

Ice consumption causes sun-burn.
Motivation

- Can one formulate physical theories without the assumption of background space-time or causal structure?
  
  Using tools of quantum information to address problems that traditionally have been considered within quantum gravity

- Quantum correlations are the crucial resource for performing computational tasks that are impossible classically.
  
  “Superpositions of quantum circuits”

Indefinite Causal Structures

Alice always measures in the z basis and encodes the bit in the z basis

Alice’s CP map: \( |z_x \rangle \langle z_x|^{A_1} \otimes |z_a \rangle \langle z_a|^{A_2} \quad x, a = \pm 1 \)

If Bob wants to receive \( b' = 1 \), he measures in the z basis

\[
W^{A_1 A_2 B_1 B_2} = \frac{1}{4} \left[ \mathbb{1} + \frac{1}{\sqrt{2}} \left( \sigma_{z}^{A_2} \sigma_{z}^{B_1} + \sigma_{z}^{A_1} \sigma_{z}^{B_2} \right) \right] \\
\langle z_\pm | \sigma_x | z_\pm \rangle = 0
\]

Channel from Alice to Bob Not seen by Bob

Bob receives the state

\[
\widetilde{W}^{B_1 B_2} = \frac{1}{2} \left( \mathbb{1} + a \frac{1}{\sqrt{2}} \sigma_{z}^{B_1} \right)
\]

He can read Alice’s bit with probability

\[
P(y = a | b' = 1) = \frac{2 + \sqrt{2}}{4}
\]
Indefinite Causal Structures

If Bob wants to send \((b' = 0)\), he measures in the \(x\) basis and encodes in the \(z\) basis conditioned on his outcome.

Bob’s CP map:

\[
|x_y\rangle\langle x_y|^{B_1} \otimes |z_{by}\rangle\langle z_{by}|^{B_2}
\]


\[
W^{A_1 A_2 B_1 B_2} = \frac{1}{4} \left[ 1 + \frac{1}{\sqrt{2}} \left( \sigma^A_{z} \otimes \sigma^B_{z} - \sigma^A_{z} \sigma^B_{z} \right) \right] \\
\langle x_\pm | \sigma_z | x_\pm \rangle = 0
\]

Not seen by Bob

Channel from Bob to Alice, correlated with Bob’s outcome

Alice receives the state

\[
\tilde{W}^{A_1 A_2} = \frac{1}{2} \left( 1 + b \frac{1}{\sqrt{2}} \sigma^A_{z} \right)
\]

She can read Bob’s bit with probability

\[
P(x = b | b' = 0) = \frac{2 + \sqrt{2}}{4}
\]
Indefinite Causal Structures

\[ W^{A_1 A_2 B_1 B_2} = \frac{1}{4} \left[ \mathbb{1}^{A_1 A_2 B_1 B_2} + \frac{1}{\sqrt{2}} \left( \sigma_z^{A_2} \sigma_z^{B_1} + \sigma_z^{A_1} \sigma_x^{B_1} \sigma_z^{B_2} \right) \right] \]

There are quantum processes that **cannot** be understood as probabilistic mixtures of causally ordered situations!
Artistic view on `Superpositions of causal orders‘ inspired by M. C. Escher’s *Ascending and Descending*, 1960

Jonas Schmöle, University of Vienna
Where to look for non-causal processes?

1. **Within standard quantum mechanics?** It may be possible to create causally non-separable situations similarly to mixtures, making clever use of superposition and entanglement.

2. **Closed time-like curves?** Every $W$ can be seen as a noisy channel back in time.

   Grandfather paradox is avoided.

   This CTC-like structure is linear, unlike Deutsch's or Bennet's CTC models.
Where to look for non-causal processes?

3. “Superposition of causal orders”? Every $W$ contains terms that correspond to situations with definite causal order, yet is not a classical mixture of those.

[Not shown]: In the classical limit all processes are causally separable, i.e., global causal order arises!

Space-time may emerge in a quantum-to-classical transition.