Pre- and post-selection, weak measurements, dynamical nonlocality & the flow of time in Quantum Mechanics

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Time in quantum mechanics: 2 mysteries

Nonlocality
How can a single particle be at 2 places at the same time?
Time in quantum mechanics: 2 mysteries

**Nonlocality**

How can a single particle be at 2 places at the same time?

**Indeterminacy**

Einstein: “God does not play dice.”
Kinematic nonlocality: Einstein-Podolsky-Rosen

Courtesy Popescu
Kinematic nonlocality: Einstein-Podolsky-Rosen

Courtesy Popescu
Indeterminancy: playing dice

- 1\textsuperscript{st} atom decays in 1 min

- 2\textsuperscript{nd} atom (identical to 1\textsuperscript{st} atom) decays in 1 hour

- There was no difference between them in the beginning, but they behave differently later
Boundary conditions: classical vs quantum

- Classical (dependent):
  \[(X,P)_{\text{init}} \rightarrow (X,P)_{\text{fin}}\]

- Quantum (independent):
  \[|\Psi\rangle \rightarrow \langle \Phi | \langle \Phi | \langle \Phi |\]
Standard formulation of quantum theory

• the state of a system at a given moment is described by one wave-function, evolving from the past to the future

\[ |\Psi\rangle = e^{-i \int_{t_1}^{t} H_{\text{FREE}} dt} |\Psi'\rangle \]

Courtesy Vaidman
Boundary conditions: quantum

What can we say about the system at intermediate time, $t$ by using strong measurements?

$$Pr(a_j, t \mid \Psi, t_1; \Phi, t_2) = \frac{\left| \langle \Phi \mid U_{t \rightarrow t_2} a_j \rangle \langle a_j \mid U_{t_1 \rightarrow t} \Psi \rangle \right|^2}{\sum_n \left| \langle \Phi \mid U_{t \rightarrow t_2} a_n \rangle \langle a_n \mid U_{t_1 \rightarrow t} \Psi \rangle \right|^2}$$

Boundary conditions: quantum

What can we say about the system at intermediate time, $t$?

Strong measurement: Aharonov-Bergmann-Lebowitz (ABL) formula

$$Pr(a_j, t|\Psi, t_1; \Phi, t_2) = \frac{|\langle U_{t_2\rightarrow t}\Phi |a_j\rangle\langle a_j |U_{t_1\rightarrow t}\Psi \rangle|^2}{\sum_n |\langle U_{t_2\rightarrow t}\Phi |a_n\rangle\langle a_n |U_{t_1\rightarrow t}\Psi \rangle|^2}$$

Time-symmetric (or 2-vector) re-formulation of quantum mechanics

- The state of a system at a given moment is described by two wavefunctions, one evolving from the past to the future, and one evolving from the future to the past.

- “Collapse” does not necessarily imply arrow of time at microscopic level.
Strong measurements
To be useful and interesting, any re-formulation of QM should meet several criteria, for example:

1) TSQM is consistent with standard QM

Inventing new theory requires ‘imagination in a terrible strait-jacket’ Feynman

\[ \langle \Psi_2 | U_{t \rightarrow t_2} = \langle U_{t \rightarrow t_2}^\dagger \Psi_2 | = \langle U_{t_2 \rightarrow t} \Psi_2 | \]

\[ U_{t \rightarrow t_2}^\dagger = \{ e^{-iH(t_2-t)} \}^\dagger = e^{iH(t_2-t)} = e^{-iH(t-t_2)} = U_{t_2 \rightarrow t} \]

2) TSQM brings out features in QM that were missed before (e.g. weak values)

3) TSQM lead to simplifications in calculations and stimulated discoveries in other fields

4) TSQM suggests generalizations of QM
A time-symmetric formulation of quantum mechanics

Yakir Aharonov, Sandu Popescu, and Jeff Tollaksen

Quantum mechanics allows one to independently select both the initial and final states of a single system. Such pre- and postselection reveals novel effects that challenge our ideas about what time is and how it flows.

*Physics Today*, November 2010

See also 6 reply letters in May 2011 + replies to come
Weak measurements
Weak values

\[ A_w = \frac{\langle \Phi | A | \Psi \rangle}{\langle \Phi | \Psi \rangle} \]

At any intermediate time, such a particle has well-defined values of the two noncommuting spin components $S_z = +\frac{1}{2}$ and $S_x = +\frac{1}{2}$ along the $45^\circ$ direction in the $zx$ plane. These values would have to be $\pm \frac{\sqrt{2}}{2}$, an impermissible value.

The spin component $S_z$ along the $45^\circ$ direction can be expressed as $\hat{S}_{45} = \hat{S}_z + \hat{S}_x$. Hence, $S_{45} = \frac{\hat{S}_z + \hat{S}_x}{\sqrt{2}} = \frac{1}{2} + \frac{1}{2}$?
Example: spin

Measuring both the x and z spin components of a spin-1/2 particle in the interval between the pre- and postselections yields different results depending on the time order of the two measurements.

- If the 1st interim measurement is along z & the 2nd is along x, both measurements are determined by the selections.
- Reversing the order of the interim measurements destroys that certainty.

\[ |\hat{S}_x = +\frac{1}{2} \rangle \]

\[ \hat{S}_x = +\frac{1}{2} \quad \hat{S}_z = +\frac{1}{2} \quad |\hat{S}_z = +\frac{1}{2} \rangle \]

\[ |\hat{S}_x = +\frac{1}{2} \rangle \]

\[ \hat{S}_z = \pm \frac{1}{2} \quad |\hat{S}_z = +\frac{1}{2} \rangle \]
Eccentric Weak Values

\[ |\hat{S}_x = +\frac{1}{2}\rangle \]

\[ |\hat{S}_z = +\frac{1}{2}\rangle \]

\[ (S_{\frac{\pi}{4}})_w = \frac{\hat{S}_x + \hat{S}_z}{\sqrt{2}} = \frac{1}{2} + \frac{1}{2} = \frac{\sqrt{2}}{2} \]

\( \sqrt{2} \times \text{times bigger than allowed} \)
Other experimental verifications:


For weak value resolution of Hardy’s paradox

Aharonov, Botero, Popescu, Reznik, Tollaksen, PLA, v301, 130 concerning Hardy’s paradox
Quantum ‘Miracles’

- ‘Well I've often seen a cat without a grin,” thought Alice “but a grin without a cat! It’s the most curious thing I ever saw in all my life.

JT, 2001; Aharonov & Reznik, 2005
Weak Values and Contextuality

- Proof that pre-and-post-selected QM is contextual
- Weak value signature that can be tested experimentally
- New restrictions on hidden variables

Weak values and causality: game of errors

- Probability distributions for different outcomes of the measurement of the spin component $S_{45^\circ}$ of a system of $N$ spin-$1/2$ particles preselected in the state $|S_z = N/2\rangle$

- Green represents measurement outcomes for an ideal measurement without postselection

- Blue curve represents probabilities in a weak measurement

- After postselection for $|S_x = N/2\rangle$, only the red distribution way out in the tail survives

Probability to obtain weak value as an error of the measuring device is greater than the probability to obtain weak value
Weak values and causality: game of errors

Consider weak measurements without postselection
Weak values and causality: game of errors

\[ \langle \Psi | A | \Psi \rangle = \sum_i P(\Phi_i | \Psi ) \frac{\langle \Phi_i | A | \Psi \rangle}{\langle \Phi_i | \Psi \rangle} \]

Weak value sum rule
Weak values and causality: game of errors

- Can always constructively interfere tails of measuring device to reconstruct initial WF of MD around the forbidden weak value

Superposition of positive shifts yields negative shift

Time-Symmetric formulation of QM (TSQM)

To be useful and interesting, any re-formulation of QM should meet several criteria, for example:

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2) TSQM brings out features in QM that were missed before (e.g. weak values)
3) TSQM lead to simplifications in calculations and stimulated discoveries in other fields
4) TSQM suggests generalizations of QM
New paradigm for amplifying signals (AAV effect)

- Enhanced sensitivity by $10^4$ (Hosten, Kwiat, Science 2/8/08)

See also “Robust weak measurement “ JT JOP, (2007)
New paradigm for amplifying signals—deflection

The beam would move by a micron traveling across the US

See Dixon, Starling, Jordan, Howell, PRL, 102, 173601 (2009)+many other articles

Courtesy Jordan
Multiple-time states & measurements
Multiple-time measurements & operators

• Whenever we consider multiple instants of time, the most general object is any combination of bras & kets

\[ \sigma_x(t_1) - \sigma_x(t_2) \]

• This is an observable that gives the value zero in the case when the x-component of the spin is the same at the two times, but doesn't offer any information about the actual value of the x-component

• These are entangled states; measurements are collections of bras & kets; very similar to states

• kinematic & dynamical descriptions are united

• multi-time state is covariant
Multiple-time states & measurements
Separation of physical scales
Separation of physical scales

QUANTUM

CLASSICAL

0.000 000 000 1m

100000000000 lyears
Correspondence principle: micro->macro

Courtesy Thaller
Correspondence principle: micro->macro
New understanding of relationship between micro & macro

- Interferometer with movable mirror

- Classical and quantum explanations provide conflicting predictions regarding the change of pressure on the small mirror during fluctuation of detected intensity

Aharonov, Botero, Nussinov, Popescu, JT “On the classical limit of quantum optics”
3) stimulates discoveries in other fields

- Another example is the Quantum Random Walk (Aharonov, Davidovich and Zagury, *Physical Review A* 1993)

Time-Symmetric formulation of QM (TSQM)

To be useful and interesting, any re-formulation of QM should meet several criteria, for example:

1) TSQM is consistent with standard QM
   - YES

2) TSQM brings out features in QM that were missed before (e.g. weak values)
   - YES

3) TSQM lead to simplifications in calculations and stimulated discoveries in other fields
   - YES

4) TSQM suggests generalizations of QM
   - YES
Does the universe have a destiny?

Forward evolving branch of the universal wave function does not describe all we should know about a world.

The backward evolving (destiny) state has to be added.

Collaboration w/ Paul Davies

Courtesy Vaidman

Does the universe have a destiny?

“One of us is in trouble”
Each moment of time is a new universe

Heraclitus vs Parmenides “You never bath twice in the same river.”

Aharonov, JT, 2010, Visions of Discovery; Aharonov, Popescu, JT, forthcoming
QM Generalization: Each moment of time a new universe

- New ability to obtain a post-selected state of one particle that is completely correlated to the pre-selected state of a second particle:

- stack N particles on top of another along the time axis:
Each moment of time is a new universe

“block universe on steroids” David Albert

Aharonov, JT, 2010, Visions of Discovery; Aharonov, Popescu, JT, forthcoming
Newton’s laws: local equations of motion

In classical physics, the force has to act in the same place where the particle is
“Testing Aharonov-Bohm effects”
Nonlocality: kinematic vs. dynamic

- Bell-inequality violations follow from the Hilbert-space structure of quantum mechanics; they are purely kinematic.
- Aharonov-Bohm effect demonstrates dynamical non-locality, i.e. in the quantum equations of motion.
From perspective of a single particle, how does opening or closing the left slit affect a particle that goes through the right slit?

Feynman: “Nobody knows how it can be like that”

- using $H = \frac{p^2}{2m} + V(x)$ and $e^{i\hat{p}D}V(x)e^{-i\hat{p}D} = V(x+D)$, we find non-local Heisenberg equations of motion for modular variables:

$$\frac{d}{dt} e^{i\hat{p}D} = i\frac{\hbar}{\hbar}[H, e^{i\hat{p}D}] = i\frac{\hbar}{\hbar}[V(x) - V(x + D)]e^{i\hat{p}D}$$

- no classical counterpart:

$$\frac{df(p)}{dt} = \{f(p), H\}_{PB}$$

$$= -\frac{\partial f}{\partial p} \frac{\partial H}{\partial x} + \frac{\partial f}{\partial x} \frac{\partial H}{\partial p} \overset{0}{=} 0$$

i.e. $f(p)$ changes only if $\frac{\partial V}{\partial x} \neq 0$ at the particle’s location.

See also Nature Physics, March, 2010, Popescu
Having your cake & eating it—measuring nonlocal interactions w/o violating causality w/ pre- & post-selection, weak measurements

Post-select an eigenstate of parity
\[ \frac{1}{\sqrt{2}} \{ |\psi_L\rangle + |\psi_R\rangle \} \]

Open or close slit

Weakly measure modular variable \( \exp(ipD/h) \)
(use parity for special case of \( \psi_R + \psi_L \) and \( \psi_R - \psi_L \))

Pre-select all particles at right slit \( |\psi_R\rangle \)

Time and axioms of quantum mechanics

Why does God play dice?

Traditional answer: nature is capricious
Time and axioms of quantum mechanics

Why does God play dice?

Quantum Mechanics

↓

Nonlocality  ☽  Relativity

• Not intuitive: “It’s like trying to derive special relativity from the wrong axioms.” – Yakir Aharonov

• This recalls a Woody Allen joke:

  This guy goes to a psychiatrist and says, “Doc, my brother’s crazy – he thinks he’s a chicken!”

  The doctor says, “Well, why don’t you turn him in?”

  The guy says, “I would, but I need the eggs!”

• We say, “Quantum theory is crazy – but we need the eggs!”
Time and axioms of quantum mechanics

Quantum Mechanics

Nonlocality

Relativity

Courtesy Popescu
New intuition about nature of time

Nonlocality ⊗ Relativity

Quantum Mechanics

Courtesy Popescu
Conclusions

1) TSQM is consistent with QM

2) TSQM brings out features in QM that were missed before (e.g. weak values)
   • if your only tool is a hammer, then you tend to treat everything as if it were a nail
   • To grasp the world more fully by grasping it gently

3) TSQM lead to simplifications in calculations & stimulated discoveries in other fields

4) TSQM suggests generalizations of QM

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