Looking at the World Sideways

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1. The Problem of Time

**Manifest Time**
- Shared Now
- Updating Past, Present, Future
- Direction
- Flow
- Indepedent of Matter
- Productive
- Etc

**Physical Time**
Many theories, so many temporal structures...

**Example:** \(<M, g, B, \text{sim}>\)

but all of them lack many central features of MT.

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**The Problem:** "Two" times need to be reconciled

**Compare with Eddington’s "Two Tables":**

**Macro**
- Solid
- Substantial
- Rigid
- Brown

**Micro**
- Empty space, mostly
- Uncolored
- Not rigid, solid, etc.

**And try to tell similar reconciliation story...**
2. **Example: Classical Time**

![Diagram](image)

Betweenness relation $B(a, c, b)$

Time is linearly ordered quotient set corresponding to simultaneously equivalent relation: $\langle M', g, B, \sim \rangle$

No now, no tensed impartial structure, no flow, no direction, etc.

But: If a foliation is a possible necessary condition for some of these features, then classical physics does have a distinguished one.

Not friendly to MT, but could be worse...

But: "Friendliness" depends on property and formulation. If independence from matter distribution is part of MT, and one considers Cartan formulation, where $\nabla^2 q = 4\pi G \rho$ goes to

$\rho_{ab} = 4\pi G \rho a_{tb}$, then matter distribution affects the ticking of the master clock.
3. The Threat from Relativity (Well-known)

Two temporal structures:

Coordinate time: in Minkowski spacetime, "natural" coordinate of an inertial observer who takes herself to be at rest.

\[ \langle M', B \rangle \sim \text{no inertial features} \]

Proper time:

\[ \tau = \int_p (g_{\mu\nu} \, dx^\mu \, dx^\nu) \, dp \]

\[ \{ \{ \{ \} \} \} \]

Clock hypothesis: ideal clock measures \( \tau \)

\[ \langle M', g, B \rangle \]

Do either of these times fit with MT?

- Not even close

Me You Andromeda

Is event A in the Past or Future?

Putnam-style argument:

- Weingard
- Stein
- Callander
- Savitt

Upshot: would need to add structure
4. A "Markovian" or "productive" picture of the world

3-dimensional spatial objects and slices marching forward, screening off previous states and determining later ones.

Philosophers:
- Maudlin: "The passage of time underwrites claims about one state "coming out of" or "being produced by" another, while a generic spatial... asymmetry would not underwrite such locations."
- Tooten: "The direction of time is direction of causation."

"next"

PRODUCES
IN RELATIVITY, DESPITE ALL THE CHALLENGES TO MT, A "PRODUCTIVE" PICTURE SEEMS LIKE IT CAN BE MAINTAINED...

—WELL, SO LONG AS WE IGNORE SOLUTIONS WHERE IT CAN'T BE, E.G., GÖDEL SPACETIME

indeed, the non plus ultra of production even sometimes holds, i.e., a well-posed Cauchy problem

\[ a. \text{ existence of solutions} \\
b. \text{ uniqueness of solutions} \\
c. \text{ continuous dependence} \]

3+1

\[ (h, K) \]

\[ \text{hyperbolic evolution} \]

\[ \text{"elliptic" constraint (h, K)} \]

THANKS TO THEOREMS PROVED IN THE 1970's BY GEROCH, CHOQUET-BRUNAT, AND OTHERS, WE KNOW INTIMATE LINKS AMONG

GLOBAL HYPERBOLICITY ~ TIME FN ~ WELL-POSED CAUCHY PROBLEM
6. Characteristic Initial Value Problem

Sachs 1962, Penrose 1963, Dautcourt 1963, and others... Geroch, Smallwood, D'Inverno, Statchel, Rendall, Luk...

Philosophers of time haven't examined this case. My present interest - can we get "production" arrow and mt-arrow to misalign?

\[ \Uparrow \leftrightarrow \text{Production} \]

Time

Null IVP: "initial" data on lightlike surface

- used in numerical relativity, holographic principle, in problems concerned with radiation, quantum gravity, and elsewhere

Answer: yes!
7. CHARACTERISTIC, CONT.

"2+2" is perhaps most popular, most results...

Intuitively, the misaligned arrows...

A \rightarrow B "takes time"

E.g. think flashlight, turning on at A "produces" B

But well-posed problem tells us wedge 1 produces wedge 2.
IS NULL-LIKE CASE AN UNNATURAL GROSSLY TWINING OF SPACE-LIKE CASE?

- Lots of well-posed problems
- Initial data = geometrical invariants w/ natural interpretation
- Can be set in spacetimes where space-like case can't be
- Closer to cosmological practice (somewhat)
- Best of all, in a well-defined sense, more natural because nasty initial constraints go away (or reduce to ODE's that are comparatively easy)

Of course, problems with caustics, etc., but problems, limitations in spacelike case too.
9. TIMELIKE "INITIAL" VALUE PROBLEMS?

CAN WE GO EVEN FURTHER + PLACE DATA ON MIXED (+-+) 3-SURFACE + "EVOLVE" SIDEWAYS INTO A SPACELIKE DIRECTION (+)?

THAT WOULD BE THE ULTIMATE MISALIGNMENT BETWEEN PRODUCTION + TIME DIRECTIONS

SHORT ANSWER: MAYBE OPEN PROBLEM

NO? Courant + Hilbert + Tarski ... IF second order linear PDE (lot of physics)

THEN NO if data on (+-) surface

YES? Craig + Wenslau: IF constraints satisfied, THEN YES for free wave eq

Geroch conjecture: IF constraints satisfied, THEN YES in KG + GR
10. Conclusion: Win, Win

If no...

Then we learn an important and deep difference between time + space, even in relativity (related to metric signature ASY).

Time* is then the direction in which we can tell the most powerful "stories".

* Or time and null...

If yes...

Then messy picture of production (in sense here):

But time might still emerge as best balance of power + simplicity...