Antigravitation

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What do I mean with Anti-gravitating matter

- A second copy of the standard model, identical to the one we know, except for its gravitational interaction.
- Both sorts of particles interact only gravitationally.
- In particular, anti-matter has completely normal gravitational properties.
- Disclaimer: This talk is unquantized.
How do we get the funny stuff to move differently?

- Covariant curves are defined via a connection. Yet this connection is unique only after requiring it to be torsion-free and metric-compatible.
- Need second derivative, throw out metric-compatibility.
- Introduce instead second metric $h$ with which the second connection $h\nabla$ is compatible.

\[(g)\nabla, (g)\mathcal{R} \quad \text{with} \quad (g)\nabla g = 0\]
\[(h)\nabla, (h)\mathcal{R} \quad \text{with} \quad (h)\nabla h = 0\]
The Pullovers

- 2nd metric provides another interpretation of the manifold (same manifold, different distance measures) and results in different curves for particles. Its local coordinate basis doesn’t normally coincide with ours.
- Two sorts of indices: $\nu$ raised/lowered with $g$, $\nu$ raised/lowered with $h$
- Pullovers to identify $h$-fields with observables for $g$-observer and vice versa: $P_g, P_h$. Locally linear maps on tensor algebras.
- $h$ is then related to a two-tensor $h = P_h(h)$ and vice versa $g = P_g(g)$.
- Induces pulled-over derivatives by metric-compatibility:

$$P_h((h)\nabla A) = (h)\nabla P_h(A)$$
$$P_g((g)\nabla A) = (g)\nabla P_g(A)$$
Equations of Motion for Matter Fields

- Action for field

\[ S = \int d^4 x \sqrt{-h} \, P_h \left( h^{\nu\kappa} (h) \nabla_\kappa \phi(h) \nabla_\nu \phi \right) \]

- Leads to

\[ P_h \left( (h) \nabla^\alpha(h) \nabla_{\alpha} \phi \right) = 0 \]

Same as

\[ (h) \nabla^\alpha(h) \nabla_{\alpha} P_h(\phi) = 0 \]

Same as

\[ (h) \nabla^\alpha(h) \nabla_{\alpha} \phi = 0 \]
How to determine the second metric?

- For convenience

\[ g_{\varepsilon\lambda} = a^\varepsilon_{\lambda} h_{\nu\kappa} , \quad a^\varepsilon_{\lambda} = [P_g]^\varepsilon_\varepsilon [P_h]^\varepsilon_\nu \]
\[ g_{\varepsilon\lambda} = a^\varepsilon_{\lambda} h_{\nu\kappa} , \quad g_{\varepsilon\lambda} = a^\varepsilon_{\lambda} a^\nu_{\nu\kappa} , \quad h_{\nu\kappa} = a^\varepsilon_{\nu} a^\varepsilon_{\kappa} \]

- The \( a \)'s are not independent: \( \delta a^{\nu\kappa} = \delta a^{\nu\kappa} = 0 \).

- Now use symmetry principle

\[ (g) R_{\kappa\nu} - \frac{1}{2} g_{\kappa\nu} (g) R = T_{\kappa\nu} - |P_h| \sqrt{\frac{h}{g}} a^\nu_{\nu} a^\kappa_{\kappa} T_{\nu\kappa} \]
\[ (h) R_{\nu\kappa} - \frac{1}{2} h_{\nu\kappa} (h) R = T_{\nu\kappa} - |P_g| \sqrt{\frac{g}{h}} a^\kappa_{\kappa} a^\nu_{\nu} T_{\kappa\nu} \]

with

\[ T_{\mu\nu} = - \frac{1}{\sqrt{-g}} \frac{\delta L}{\delta g^{\mu\nu}} + \frac{1}{2} g_{\mu\nu} L , \quad T_{\nu\kappa} = - \frac{1}{\sqrt{-h}} \frac{\delta L}{\delta h^{\nu\kappa}} + \frac{1}{2} h_{\nu\kappa} L \]

- Degrees of freedom in pull-overs needed to fulfill Bianchi identities.
Action

- Full action

\[
S = \int d^4x \sqrt{-g} \left( (g) R/8\pi G + \mathcal{L}(\psi) \right) + \sqrt{-h} P_h(\mathcal{L}(\phi)) \\
+ \int d^4x \sqrt{-h} \left( (h) R/8\pi G + \mathcal{L}(\phi) \right) + \sqrt{-g} P_g(\mathcal{L}(\psi))
\]

- Dynamical variables \( g \) and \( h \), \( \psi \) and \( \phi \).

- Variation of \( g_{\epsilon\lambda} h_{\kappa\nu} a^{\epsilon\kappa} a^{\mu\nu} = \delta^\mu_\lambda \) with \( \delta a^{\epsilon\kappa} = 0 \) yields

\[
\delta h_{\kappa\lambda} = -[a^{-1}]^\mu_\kappa [a^{-1}]^\nu_\lambda \delta g_{\mu\nu}
\]

- Note: There is no negative kinetic energy in the action. Variation over fields (for ‘inertial’ stress-energy) has no change of sign. Sign change only for source term of field equations.

- Both gravitational AND inertial mass (energy) are conserved, thus no vacuum decay possible.