Beyond Math

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Abstract: In this essay I reflect on the use and usefulness of mathematics from the perspective of a pragmatic physicist. I first classify the different ways we presently think of the relation between our observations and mathematics. Then I explain how we can do physics without using math – that we are in fact already doing it. In the end the pragmatic reader will know why math is reasonably effective, why we are all models, and how to go beyond math.

1. The pragmatic physicist

Once upon a time "universe" meant all there is. But now that physicists have several variants of multiverses the universe isn't what it used to be, and unfortunately no two people agree exactly what the multiverse is either. Latin lovers all over the world are grinding their teeth but it is rather pointless to insist using a word according to its etymology if this just results in communication failure. I will therefore refer to the all-there-is as "All."

For the purpose of this essay I take on the perspective of a strictly pragmatic physicist. The pragmatic physicist – first name Pragmatic, last name Physicist – wants to describe observations and only bothers to think if thinking seems useful for this description.

Pragmatic Physicist doesn't have reason to believe that humans are unique and their observations are special. If he or she[1] refers to observations in general, he thus means all observations that could be done by anybody or anything anywhere at any time. We denote these observations with capital O. If he refers only to observations made by humans so far, we denote these as a small o.

Pragmatic Physicist is interested in the All only to the extent that it concerns either an observation or a tool to describe an observation. He does not know what it might possibly mean for something to "exist" or to "be real" if it can't be observed and not be used for anything, but then he doesn't care enough to deny its existence either. Even leaving aside superstition and religion, the belief that human consciousness has a non-physical component is widely spread. Pragmatic Physicist rolls his eyes upon such romantic interpretations of synapses and neurons, but since it doesn't matter to him, he just ignores the question whether the All is more than that what can be observed and what is being used to describe the observations.

This means we will not discuss here the question what is real and what this might possibly mean, because the pragmatic physicist doesn't care. We will only discuss observations and their description. You are free to believe there is another part of the All and call the pragmatic physicist a soulless moron.

The pragmatic physicist is not a follower of the shut-up-and-calculate doctrine. He knows that usefulness depends on both context and time, and he thinks it is shortsighted to just dismiss philosophy. Like the CEO who has an eye on customers too young to have purchasing power, Pragmatic Physicist has an eye on philosophers' discourse so he is ready if they become useful. If you push him, he admits he can't name any philosopher who makes sense to him, but he also knows that he knows nothing and vaguely recalls some philosopher figured that out a long time ago.

Pragmatic Physicist's view on science is shaped by what he learned as student. The purpose of scientific inquiry in his opinion is to develop models that explain observations. Once a model is found it can often be used to manipulate nature to better suit human needs. This is only possible if the same

[1] In the hope that the female reader will forgive me, I henceforth use the male article.

model is suitable for similar but different subsystems of the observable part of the All. With 'subsystem' Pragmatist loosely means anything that is not the whole (of all observations). These subsystems often fulfill additional properties because otherwise it becomes very difficult to find models, but for our purposes we will not have to specify these properties.

Science works by identifying a model that can be mapped to subsystems of what Lee Smolin likes to call the "Real World Out There" [1] and what Pragmatist refers to as the totality of all possible observations, O. He doesn't really know how to explain operationally what an observation is, but he recognizes one if he sees it. If you insist, he would say that an observation has taken place if one subsystem has obtained information about another subsystem.

In modern physics, models are mathematical. The difference between pure mathematics and physics (and some other parts of the natural sciences) is that a physical theory does not consist solely of mathematics, it also must contain a prescription to identify the mathematical structure with observation. Pragmatist was taught that science is all about finding regularities. These regularities are what the mathematical models capture, and their understanding and subsequent application is what makes science so useful.

To be useful, a theory must do more than reproduce observations on one subsystem – this could be done already by recording the observation. To be useful, a theory must provide an advantage over just waiting and seeing what happens. It must either make a prediction (about future observations, not necessarily about future events), or succeed in describing many observations by the same explanation.

Pragmatist is aware that he does not use the word "theory" to mean what the US American popular science media tries to convince its readership it means, namely that scientists who say "theory" mean a model that is mapped to observation which has already been found to be correct to high accuracy. The reason he isn't using the word this way is that as a matter of fact it's not how physicists use it, though string theorists certainly wish it was. He is too pragmatic to worry that some Creationist will read an FQXi essay, so Pragmatist just asks you to understand he uses the word "theory" to mean a model including a map to observations, regardless of whether this theory has already been shown to be useful.

2. The mathematical universe

Pragmatic Physicist has read Max Tegmark's paper "The Mathematical Universe" [2]. Tegmark argues that the All is identical to mathematics. Pragmatic finds that mildly interesting but useless and forgets about Tegmark until he comes across Sabine Hossenfelder's blog. Hossenfelder explains she dislikes Tegmark's Mathematical Universe because it unimaginatively assumes that mathematics is the best way to describe observation, rather than just the best way humans have found so far [3].

Now Pragmatist worries. What if Hossenfelder is right and there are other, possibly better, ways to describe observations than using math? What if some of the observations we presently have cannot be described by math at all? What could possibly be better than math? He cannot imagine. Neither, for that matter, can Hossenfelder as she admits in her writing.

On his birthday Pragmatic logs in for his annual facebook visit to upthumb the birthday greetings. He doesn't really know what facebook is good for, but he quite likes the video of the baby armadillo and some hours later he finds himself reading Mark Twain's hilarious essay about "The Awful German Language" [4]. Twain writes "When a German gets his hands on an adjective, he declines it, and keeps on declining it until the common sense is all declined out of it. It is as bad as Latin."

In a flash of insight it occurs to Pragmatic that he just learned something about a language he doesn't speak by reading about it in a different language. Indeed, he now realizes that one could write essays

about mathematics using plain old English! While one will never learn German without using German vocabulary, and never learn mathematics without using equations, one can still learn something about one language using a different language.

And so Pragmatic Physicist takes his mathematical toolbox and sets out to apply it to the question whether all observations can be described by math.

3. A mathological classification

Pragmatic physicist reasons: Either Tegmark is right and observations can be mathematics rather than just being described by mathematics, or Tegmark is wrong and this isn't so. In the first case, either all observations are math, or not all. In the latter case, either all observations can be described by math or not. We denote with M the entirety of all mathematics[2] and with O all observations. We know that there exist observations that can be described by math, so if observations are not themselves math there exists some map, T, that maps the models, at least some of which are math, to observations. This leads to the following four cases:

1. Observations can be math and all observations are math $\{\mathrm{O}\}\cap\{\mathrm{M}\} \neq \{\emptyset\}\wedge\{\mathrm{O}\}\cap\{\mathrm{M}\} = \{\mathrm{O}\}$
2. Observations can be math and not all of them are math $\{\mathrm{O}\}\cap\{\mathrm{M}\} \neq \{\emptyset\}\wedge\{\mathrm{O}\}\cap\{\mathrm{M}\} \neq \{\mathrm{O}\}$
3. Observations are described by math but are not math and all observations can be described by math $\{\mathrm{O}\} \cap \{\mathrm{M}\} = \{\emptyset\} \wedge \{\mathrm{T(O)}\} \cup \{\mathrm{M}\} \neq \emptyset \wedge \{\mathrm{T(O)}\} \cap \{\mathrm{M}\} = \{\mathrm{T(O)}\}$
4. Observations are described by math but are not math and not all observations can be described by math $\{\mathrm{O}\} \cap \{\mathrm{M}\} = \emptyset \wedge \{\mathrm{T(O)}\} \cup \{\mathrm{M}\} \neq \{\emptyset\} \wedge \{\mathrm{T(O)}\} \cap \{\mathrm{M}\} \neq \{\mathrm{T(O)}\}$

These four cases can be illustrated in Venn diagrams (Figure 1). For clarity we have further separately depicted the cases in which subsets are identical to the mother set. We have also in the diagrams added $\{\mathrm{o}\}$, the observations that humans already have so far, which is a subset of $\{\mathrm{O}\}$, and their images $\{\mathrm{T(O)}\}$ and $\{\mathrm{T(o)}\}$. The diagrams show the situation in which some of our observations cannot be described by math, or are not math respectively. The arrows indicate how theories in physics presently operate.

In more detail:

1a. All observations are math, and there is no mathematics that cannot be observed. This is Tegmark's Mathematical Universe whose philosophy we will call Tegmarxism. Science in this case is the task of finding similar mathematical structures and maps between them.

1b. All observations are math but only some of math appears as observation. This sounds much like Garrett Lisi's assertion that yes, the universe is mathematics, but only the prettiest mathematics. We will call this the Mauritian Variant of Tegmarxism.

2a. Observations can be math but some of our observations are deprived of the mathematical ideal form. We will call this the Platonic Street View because it is what the person on the street would think if you told them that math is real.

2b. All math can be an observation, but not all observations are mathematical. This is the extended version of Tegmarxism that Hossenfelder argued for. We will refer to this as Post-Tegmarxism.

3a. All observations can be described by math and all math is used for some observation. In this case Pragmatic Physicist need not worry that his math will ever run out of explanatory power or that some awesome math book might never be put to work. We will call this Omnimathism.

[2] We will turn to the question whether M contains itself later.

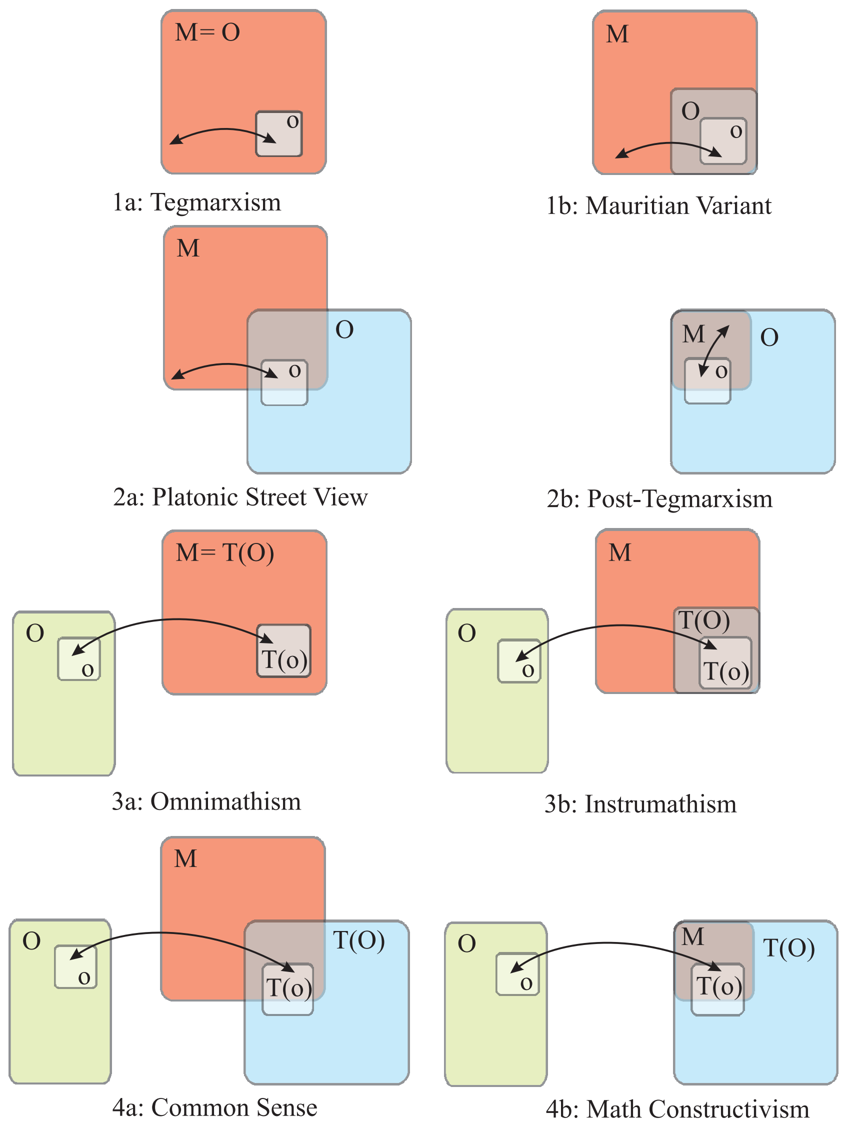

Figure 1: Classification of relations between mathematics, M, and observations, O. Which one is your philosophy?

3b. All observations can be described by math, but not all math is necessary. In this case mathematics is the instrument scientists can use to describe all of nature. We will call this Instrumathism.

4a. Observations are described by math but are not math. Not all observations can be described by math and not all math is useful to describe observations. This is what the layperson typically thinks of mathematics, we thus call it Common Sense.

4b. Not all observations are described by math, but there is no math that does not describe some observation. This is a quite untypical view. It is most similar to the philosophy of those arguing that mathematics is a human invention and only exists to the extent that we use it. As explained earlier, Pragmatic Physicist it not interested in discussing the meaning of existence but with a hat tip he refers to this option as Math Constructivism.

Having gotten so far, Pragmatic Physicist is somehow pleased but now he doesn't know what to do next. How can he possibly address the question which one of these cases is correct? How could he find out? And what do we do if mathematics cannot be used to describe all our observations? Searching

for inspiration, he turns to Eugene Wigner's influential essay on the "Unreasonable Efficiency of Mathematics in the Natural Sciences" [5].

4. The role of mathematics in physics

Pragmatic Physicist is not concerned with the question what mathematics "is". He doesn't even know what that question means. He is only concerned with the question what mathematics is good for. Mathematics is often described as a language or a tool, which is also how Wigner refers to the role of mathematics in physics. The difference to other languages and tools – the relevant difference that makes math so dramatically useful – is that it is entirely self-referential. Or, as Tegmark put it, "free of human baggage."

The languages that humans evolved for communication refer to human experiences, and the meaning of these words depends on context. The same word can mean two entirely different things depending on the situation or even facial expression, and most people consciously or unconsciously adapt their language to local customs. Human language is efficient to connect with others around us, not to be precise and reproducible. If you insist on the correct pronunciation of "déjà vu" all that people will understand is that you think they're dumb for not speaking French.

Words are malleable, they can be played, they can be abused. Languages evolve and adapt. Writers make a living from recreating language over and over again and we admire the novelty. But the very reason that makes language socially useful makes it so unsuitable for science. It's imprecise, open to interpretation, dependent on a large number of unrecorded factors. As they say, if a thousand people read a book, they read a thousand different stories.

Mathematics on the other hand does not suffer from these shortcomings. Mathematical structures are defined to have certain properties. They're not open to interpretation and don't depend on the context. Mathematics is therefore highly reproducible and precise. If a thousand people read Einstein's field equations, they read the same equations.

There is nothing unreasonable about the efficiency of mathematics as a language because we call it efficient exactly because it is reproducible and precise. The actual surprise, as Wigner discusses in his essay, is that we find ourselves in an environment that has allowed us to discover many laws which lend themselves to a mathematical formulation. "The laws of nature," Wigner writes, "must already be formulated in mathematical language." Really? Pragmatic raises an eyebrow. What we actually know is that some laws of nature can be formulated in mathematical language. But does this mean our observations must be describable as mathematical law?

Pragmatic Physicists thinks of his ancestor, Pragmatic Neanderthal. She climbed to the top of the fire mountain and scanned the land to her feet. "Why is the human eye so effective at perceiving the all-there-is?" she mumbled to herself. "It is not," said the time-traveler who just appeared behind her, "It is due to natural selection that the surviving species are likely to be good at perceiving what is relevant to their survival. You do not in fact perceive most of the all-there-is." He was in the middle of explaining galactic rotation curves when she hit him over the head and sacrificed him to the Fire God. How effective is your math at describing her behavior?

Wigner carefully wrote about the efficiency *in the natural sciences*. But what, fundamentally, is not part of nature? If we draw the line between the natural sciences and the 'soft' sciences based on the extent to which they use math, then it is tautologically correct that mathematics is effective in the natural sciences. Nothing can be learned about the general efficiency of math if we only look at that what we describe with it, so let us look then at that what we do not describe by mathematics.

We do not, for example, formulate studies in the history of, say, 16th century Hip-Hop culture in

mathematical quantors. It would be highly impractical, for not to say inefficient. Practical Historian, the great-aunt of Practical Physicist shudders at the thought! Aside from statistical analyses, history and sociology is still mostly verbally interpreted because it is extremely hard to quantify all the different nuances and aspects relevant to human culture.

If reductionism is correct and all of human culture is a consequence of the fundamental interactions among elementary particles, there is still an enormous amount of factors that would have to be taken into account, and most of them are unknown to us. And so we do not presently know whether it is possible at all to construct a mathematical model of human behavior that is any better than just watching the real system to see what happens. Practical Physicists reminds you that the point of doing science was not finding just any model, but a model that is useful .

It is certainly possible in certain cases to find useful mathematical models for human behavior, so this is not to say that one should not even try to describe sociology or history by math. Many examples exist where regularities can be found and applied, such as the motion of crowds [6] or the growth of cities [7]. But at least for now this is the exception rather than the rule.

Physicists like to think that it is possible in principle to develop a mathematical model for human behavior, and Pragmatic is no exception. But for such a model “in principle” to be of any scientific value “in practice” it must be more useful than an identical copy, and Pragmatic isn’t sure at all that a mathematical model of human behavior could ever predict anything. At present, even the best computers on Earth cannot predict the folding of single proteins, something that happens in nature within milliseconds.

Practical physicist has been around for long enough to know that questioning Wigner’s supposed “unreasonable effectiveness of mathematics” is blasphemy for his colleagues. “But, but,” the readers with a PhD mumble and bounce on their chairs thinking Practicial Physicist would make good fodder for the Fire God. So let me give this to you, dear reader. Practical Physicist has no idea why we find ourselves in an environment that is configured so that it displays many recurring, reproducible, and often self-similar subsystems that can be greatly simplified thanks to properties such as cluster decomposition, scale separation, and locality. All he is saying is given that these subsystems do exist, it isn’t surprising that math is efficient at describing them because the essence of mathematics’ success is the delivery of simplified universal models that are reproducible and reusable.

This brings Practical Physicist back to the question whether all is describable by mathematics.

5. Models. Behaving. Gladly.

Reminded of the role of mathematics in physics, Practical Physicist sees a way to move on. The little arrow that is science in his diagrams is not just an arrow, it is something he does constantly, something humans have done for thousands of years. He will look at examples to see which case fits.

So he thinks of himself as student calculating the energy levels of the hydrogen atom. He thinks of Einstein calculating the deflection of light on the Sun. He thinks of the Millennium Simulation [8] numerically recreating the formation of large scale structures. He thinks of the bump in a plot that we came to refer to as the Higgs boson. He thinks of the recent Nobel Prizes, of graphene, fibre optics, and supernovaes. He thinks of dark matter, superconductors, and quantum computing. And it dooms on him that none of the cases he depicted describe how science actually works.

Practical Physicist slaps his forehead and calls himself a fool because he took for granted what he learned, that we use models made of something called mathematic that are distinct from observations. But this has never has been so. Our models are observations, like that what they describe.

Think of any scientific explanation of any observable phenomenon. Take, say, the comet on its orbit

around the sun. The process of science is not, as Practical Physicist had been taught, finding and using a map from mathematics to observation. No, the models that we use are subsystems of our universe, just like the system we want to describe (Figure 2).

The model is you doing a calculation with pen on paper. It's a computer doing the calculation for you. It's a computer running a Monte Carlo algorithm. It's your student plotting a graph. In each of these cases we map one observation to another. We take the results of a calculation – a plot, a table, a number – and match them to another observation. If you think of Friedmann's equations and their solutions, you *are* a model. Maybe not a good one, all right, but the processes in your brain do exactly what we asked a useful scientific model to do.

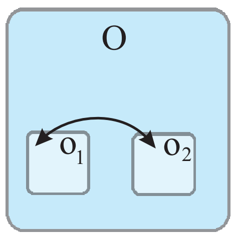

Figure 2: Science works by mapping subsytems to each other.

That complex phase of the wave-function, the prime example for a mathematical structure that is supposedly unobservable? You have seen it dozens of times. It is perfectly possible to visualize complex numbers so that humans can perceive them. That it does no longer correspond to an observable after mapping it to another subsystem is irrelevant. Nobody ever demanded that to be so. All we want of the theory (the model including the map) is to reproduce observations, all we want is for it to be useful.

6. How to do science without mathematics

Practical Physicist is really excited now because suddenly he sees how to take mathematics out of the scientific process. To be a useful model, one that makes predictions or describes many observations, the model must not necessarily be described in the language of mathematics. All we need is to know how to construct subsystems that can be mapped to other subsystems in a process that is reproducible.

If Pragmatic Neanderthal had not heartlessly sacrificed the time-traveler to the Fire God, the time-traveller might have given her a little black box with the amazing property of being able to predict volcano eruptions a month in advance. Volcanologists today would certainly like to get their hands on that little black box. But for Pragmatist, making a prediction with this box would not be science because it is not reproducible. If the box breaks, we'd be back to start.

But consider the time-traveler told you how to construct the little black box. Now you could make one yourself. You would get to know that what it is made of and you would develop an understanding for it. You could experiment with it, learn how to modify it, and how to use it for other tasks. For all practical purposes the little black boxes could be just as good as mathematical models. They could be even better because they could work in cases when mathematics does not work, if only we could give up the idea that all models have to be formulated in mathematical language.

Suppose you could initialize an adiabatic quantum computer so that it evolves into a final state from which you can read off the result of the folding of a certain protein. Quantum computers that can be constructed right now are so simple that this is far from being possible, and moreover they have never been put to use on any calculation that could not have been done also by a normal computer, using algorithms that express mathematical manipulations. But the future may be right there: In modeling nature directly, leaving out mathematics as the middle man, by directly comparing different types of subsystems.

It is not only in quantum computing that we can sense a beginning of this development. It is also implicit in the discovery and use of dualities in physics [9] and in systems that are "analogue" to each other, both of which is just a way of saying that certain observations can be mapped to each other.

These dualities have been found analyzing mathematical structures, yes, but may it be that they hold beyond that?

Take as an example Analogue Gravity [10, 11]. Analogue Gravity exploits the fact that perturbations in a fluid propagate like particles would in certain gravitational fields. The best known example is Unruh's dumb hole – a fluid in flow that creates a 'sonic horizon' for perturbations on its surface if the speed of the fluid's flow exceeds the speed by which perturbations can travel. This sounds like a curious coincidence, but it can be shown that such gravitational analogues exist in a large variety of cases. Take an imaginative leap now and suppose you could model supergalactic structure formation by observing the behavior of a suitably chosen condensed matter system in the laboratory. It would be much like running a computer simulation, just that you don't write the algorithm. You don't even need to know if one exists.

A similar statement can be made about the AdS/CFT duality [12, 13, 14]. Physicists presently use this duality as a calculation tool, but imagine you would instead create one system in the laboratory and use it to extract predictions about another system. You would not be doing a mathematical calculation, you would just map one subsystem to another.

You might argue now that the difference is one of understanding, that without understanding the use of the model is limited. Maybe that is so, but note that your idea of understanding is greatly biased by your experience that understanding can include a mathematical framework. But if a model is the only model for a specific purpose, any model is useful.

In his essay "The End of Theory" [15] Chris Anderson argued that the end of theory is nigh because computers are able to process ever increasing amounts of data and infer regularities from this data without ever developing a human-readable theory. Anderson wasn't concerned with the power of mathematics, but critics claimed that this would stall scientific progress because no true understanding is achieved, a point also relevant for the idea of science without mathematics. So let us reflect on this.

Anderson is both right and wrong. He is of course exaggerating when he says theory is coming to an end because we still need someone to write the computer code. But he is right that the relevance of skillful simplification decreases with larger computing power, and that it is ultimately unnecessary for humans to be able to formulate a simplified underlying mathematical law to do science, as long as the results are useful.

Mathematics is certainly not coming to an end. Mathematical models are and will probably remain the best way to understand observations. Pragmatic scientist does not want you to give up on math. He is just pointing out a way to still create scientific models in cases mathematics does not work.

Even if you do not believe that there are any observations for which we cannot find a mathematical model, a non-mathematical model that works by comparing observations still can be a step on the way to finding a mathematical model. You might for example have succeeded in describing the dynamics of certain fluids by means of gravity had you exploited the observed similar behavior of these systems. It's just that we had already been using the hydrodynamical equations before we discovered the similarity to gravity.

Finally, let us come back to the question whether M contains itself or doesn't. It's the classical example of a question that cannot be answered within the mathematical framework that we are using. But we already knew from the very beginning that using math to classify what a language beyond math can do would not be self-contained. Instead, think of it as a bootstrap approach to beyond-the-math-physics. The classification in Veng diagrams is a first step, but it cannot provide all answers that we would like to have. Likewise, different types of logic in which the depicted cases are not mutually exclusive constitute a generalization of this approach. Pragmatic's observation that science can proceed without necessarily using mathematics is a way to shed light on which of the philosophies

of Figure 1 is most compatible with nature. We may never be able to settle this question entirely, but we may be able to learn if there are limits to mathematics, where they are, and how close we can get to mathematical perfection.

7. Conclusions

Pragmatic Physicist hopes that you read his whole essay, but he is too pragmatic to think your reading habits are any better than his. So in case you jumped here from the abstract, a brief summary:

Scientific models must be useful to describe our observations, either by making predictions or by explaining many observations with one model. There is however no particular reason why models must be mathematical to be useful. Mathematical models today work not by identifying abstract mathematical structures with observations, but by identifying observable realizations of mathematical structures with observations. Scientific models, in general, identify different observations with each other. There is then no specific reason why this identification must necessarily involve mathematics. One could do science by directly trying to identify the observables belonging to different subsystems, and in this way learn about the effectiveness and limits of mathematics.

If technology continues to progress and flourish, the day will come when we can link human brains and language will become an unnecessary intermediary of communication. In the very same way, mathematics may one day become an unnecessary intermediary of science.

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