Of Mathematics and Radical Change: Alain Badiou's Set-Theoretical Ontology

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"In the standard positivist approach to the philosophy of science, physical theories live rent free in a Platonic heaven of ideal mathematical models. That is, a model can be arbitrarily detailed and can contain an arbitrary amount of information without affecting the universes they describe. But we are not angels, who view the universe from the outside. Instead, we and our models are both part of the universe we are describing. Thus a physical theory is self referencing, like in Gödel's theorem. One might therefore expect it to be either inconsistent or incomplete. The theories we have so far are both inconsistent and incomplete."

- Stephen Hawking, *Gödel and The End of Physics* [1]

[- Introduction -]

As scientists, when we think of "mathematics" we typically think of algebra, geometry, or calculus-- cold equations, rote calculations, and endless symbols. Certainly, if physics and the natural sciences as a whole seek to be complete and consistent, it is necessary that the mathematics serving as both its language and foundation be just as rigid. But what if this is not the case? What if the mathematical substrate that underlies all our physical theories is inherently vulnerable to outside entities that may penetrate such structures and completely transform them? These are the implications of current philosophical work being done on the development of mathematical set theory; this set theory being widely held to ground all our mathematics, and therefore having a profound influence on how our theories are structured and formalized.

Our goal in this paper will be twofold: 1) To counter the modern dogma amongst scientists that mathematics is but just another "useful language game", who's complex foundations can be disregarded in the service of "what works best" in practice, and 2) to show how a modern understanding of these foundations can inform the very mechanism by which scientific revolutions occur, and why there can be no conceivable stopping point to discovery and change.

In particular, this paper will closely follow the works of French philosopher Alain Badiou, whose body of texts has only recently begun translation into English. Our specific interests will be in his 1988 philosophical masterpiece *Being and Event*, and 1990's treatise on number theory *Number and Numbers* **[EN 0]**. While Badiou's work is grounded in the foundations of mathematics, he is nevertheless able to use this foundation to think through revolutions in such afar fields as politics, music, and art. The wager of his work is that the ability of set-theoretical mathematics to allow for novelty, revolution, and change in what are typically considered rigid and unmovable systems offers a similar model of transformation for these extra-mathematical fields of thought; that mathematics is, as the Greeks believed, the ultimate descriptor of our worlds, but that it also describes how these worlds undergo radical changes.

For us scientists, this will mean re-recognizing the primary place of mathematics in our scientific theories, and seeing in its paradoxes and transforming capabilities the very reasons *why* our now ancient dreams of consistent and complete physical formalizations are but illusions which will always give way to revolution and change. Such an appreciation can enlighten us to the true nature of physical theories, the weaknesses inherent within them, and the revolutions they undergo as a result.

We will begin with a topic usually overlooked in basic scientific education: the method by which numbers are constructed, via set theory. This will lead us directly to the outstandingly unsolved Continuum Problem, its two suggested solutions, and how these approaches affect the structure of our scientific theories. Finally, a novel (albeit brief) case study of the Einsteinian revolution from the Newtonian world will follow as a demonstration [**EN 1**].

Mathematics must once again be privileged as the immovable foundation of physics, but also understood as having the resources to allow for unpredictable and radical change.

[- **The Construction of Number** -]

From the outset of his *Number and Numbers*, Alain Badiou proposes, “A paradox: …we have at our disposal no recent, active idea of what number is… We know very well what numbers are *for*: they serve, strictly speaking, for everything, they provide a norm for All. But we still don’t know what they *are*…” [2, *italics* mine]. Lamenting the modern day role of number as that which is relegated to pure arithmetic, Badiou proposes a journey through the genealogy of the concept of number in history.

While Badiou's exposition of the history of constructing numbers from the Greeks through the great mathematicians of Frege, Dedekind, Peano, Cantor, and Conway is quite fascinating in its own right, we shall review the particular contributions of Georg Cantor to this history. The fundamental question of concern to all these thinkers was the following: Is it possible to think of numbers neither as empirical, nor as transcendental, but as *a production of thought itself*? That is, can we construct numbers on the basis of thought alone, axiomatically, and not rely on flawed conscious observations on the one hand, nor the rule of God on the other? To be able to achieve such a feat would allow for a unique realm of thought untainted by the complications of perception and the aporias of mysticism.

Cantor provides us with a novel "set-theoretical" approach to constructing numbers. Within the development of this set theory it is crucial to note that there is no explicit definition of what a set *is*, and therein lies the beauty of the conceptual edifice-- it does not rely on an *a priori* description of a set and its elements via empirical predicates. While it is intuitive to think of sets as "collections of objects", it is not necessary to define what these objects are beforehand, and these objects (or elements) may be sets themselves. All that is required is the concept of "belonging" **[EN 2]**. We should also note that “sets” may also be referred to as “multiples” (and that Badiou prefers to use this convention).

A fundamental function one can perform over any set is that of creating its “power set”, or the set made up of all the “inclusions” or “parts” of the initial set. For example, if A = {x,y,z}, the power set p(A) = {∅, x, y, z, {x,y}, {y,z}, {x,z}, {x,y,z}}, keeping in mind that an empty set is an implicit element of *every* set (see below). For all finite sets A of size n, the number of elements in p(A) will equal 2^n. The power set is a general feature of all sets **[EN 3]**.

To construct the natural numbers, one starts with the "empty set", or the set that contains *no* elements. This empty set is marked $\emptyset$. Next, one can construct a set that *contains* the empty set, by taking the power set of the empty set: $[\emptyset]$, this set containing a single element, the empty set. This process can be continued *ad infinitum* **[EN 4]**. The perceptive reader will notice that these enumerations correspond to what we know as the "ordinal" (or "natural") numbers: $\emptyset = 0$, $[\emptyset] = 1$, $[\emptyset, \{\emptyset\}] = 2$, $[\emptyset, \{\emptyset\}, \{\emptyset,\{\emptyset\}\}] = 3$, and so forth.

The genius of Cantor was to then identify a “limit ordinal”, ω_0, which represents the set of *all* these possible ordinal numbers. ω_0 is the first "infinity", and is given the "cardinality", or size, $\aleph_0$; $\aleph_0$ being the first “infinite cardinal” **[EN 5].** It suffices to describe ω_0 as the first set which, while "full" of all the ordinal numbers, does not succeed from a previous number. Just as how the empty set forms the initial basis for the construction of finite ordinals, with no predecessor of its own, ω_0 is the basis for infinte numbers, and $\aleph_0$ the basis for infinite cardinals. Badiou comments, "On condition of the existence of the void [empty set], there is 1, and 2, and 3..., all successors. But a limit ordinal? ...we find ourselves on the verge of the decision on the infinite. No hope of [empirically] *proving* the existence of a single limit ordinal. We must make the great modern declaration: the infinite exists, and, what is more, it exists in a wholly banal sense, being neither revealed (religion), nor proved (mediaeval metaphysics), but being simply decided, under the injunction of being, in the form of number... That is infinite which, not being void [or empty], meanwhile does not succeed" [3]. But, just as $\emptyset$ is succeeded by $[\emptyset]$ (or 0 by 1) with no ordinal remainder between the two, the infinite cardinal $\aleph_0$ must be succeeded by the "next" infinite size of $\aleph_1$, with no remainder in-between.

[- The Continuum Problem -]

The “Continuum Hypothesis” (hereafter “CH”) arose from the simple question of “how many points are on a line?”, or, equivalently, "how many real numbers exist?". Cantor was able to successfully demonstrate that the size of the set of real numbers (comprised of all the rational numbers, irrational numbers, and transcendental numbers) that form the linear continuum equals the size of the set of all subsets of the natural numbers (or the power set of ω_0). Thereby, Cantor believed he had solved part of the ancient continuum question-- the size of the linear continuum must equal the cardinality $2^{\aleph_0}$ **[EN** 6**]**.

Now, the crux of the Continuum Hypothesis would rest on the following equation: $2^{\aleph_0}$ *must* equal $\aleph_1$, the next possible infinite measure. The proof of such an equation would represent the totality of all real numbers with no remainder between the amount of natural numbers ($\aleph_0$) and the amount of real numbers ($\aleph_1$), thus achieving the goal of a *well-ordered, structured, universe of numbers apprehended by thought alone*. But could this equation be proven? If not, it would suggest that the size of the continuum could be identified with *any other* infinite measure. That is, all we know is $2^{\aleph_0} > \aleph_0$, but by *how much*? If $2^{\aleph_0} \neq \aleph_1$, then the continuum could not be said to have an upper limit on its cardinality, and infinite disorder would reign-- untold of possibilities of sets would be possible between the natural numbers and real numbers. $2^{\aleph_0}$ could possibly equal $\aleph_2$, $\aleph_{77}$, $\aleph_{450}$... practically *any* infinite cardinal. So, instead of binding the universe of numbers to the demands of order and structure, Cantor had unexpectedly unleashed the possibility of an infinite amount of infinities. [4][5]

Cantor's inability to prove *or* disprove his continuum hypothesis eventually drove him mad and led him to suicide. But this Pandora's Box he had opened led to the infamous "Continuum Problem" (hereafter "CP"): *Is* the size of the power set of the natural numbers-- the very continuum-- equal to the second infinite cardinal? *Is* $2^{\aleph_0} = \aleph_1$?

[- The Axiomatization of Set Theory -]

The generation of the Continuum Problem is paramount, as it is this set-theoretic universe of Cantor's that was properly axiomatized by Ersnt Zermelo and Abaraham Fraenkel in the early 1900s, forming the ZFC axiom system **[EN 7]** that provides the very foundation of mathematics; therefore of the very algebra and geometry that ground our physical sciences [6][7]. Through nine basic axioms the whole known mathematical universe could be derived. Some of these axioms we have already encountered: the Null-Set Axiom, the Axiom of Infinity, and the Power Set Axiom. The remaining axioms detail further regulations and functions of sets. As Peter Hallward comments, "The axiomatization of set theory as the foundation for mathematics completed the process begun by Descartes and the arithmetization of geometry, namely, the liberation of mathematics from all spatial or sensory intuition. Numbers and relations between numbers no longer need to be considered in terms of more primitive intuitive experiences (of objects, of nature) or logical concepts. The whole of mathematics could now be thought to rest on a foundation of its own making, grounded on its own internally consistent assertion" [8].

However, at this seemingly complete foundation still lay the Continuum Problem. And the Axiom of Foundation (one of the basic nine axioms) also houses a certain peculiarity. This axiom is a direct result of Bertrand Russell's famous paradox of self-referencing sets **[EN 8].** In order to avoid this paradox, the Axiom of Foundation states that for any non-empty set A, there is an element x belonging to A such that no sets are shared between A and x **[EN 9]**. That is, from the perspective of A there are sets belonging to the element x that are not "seen" by A. This axiom effectively outlaws any paradoxical sets belonging to themselves. However, it also creates the very real possibility of "indiscernible" elements from x; elements that are not recognized by the language of A, but are yet real. It is through this axiom that "unknown", "novel", or "repressed" elements can come to transform a situation by being incorporated into it from the "foundation". (On Badiou's important concept of "situations" see **[EN 10])**

The Axiom of Foundation effectively lays bare the same conclusions as Gödel's famous First Incompleteness Theorem: given any system which is adequate for primitive arithmetic (a "first-order predicate calculus"), there exists a predicate such that there is no complete and formal system for it [9]. The upshot of these rules is that given a well-formalized system of logical statements (such as in physics) there will always be a statement *undecidable* within that system, leaving the system either inconsistent or incomplete. The property of being undecidable is given through the language of the system; that is, the language being incapable of rendering a set as either logically "true" or "false" within the situation.

Consequently, our very use of local languages obstructs certain *predicates* or *propositions* from completing a set of theories or allowing them to consist absolutely [**EN 11**].

We shall now see how one attempt to circumvent this issue sought to subordinate all sets to a global language: that of empirical science.

[- First Approach to the Continuum Problem: Gödel's Constructible Universe -]

If we return now to CP, we will see in history two distinct attempts to solve it. Kurt Gödel provides us with the most pragmatic way of doing so: restrict the universe of sets only to those that language can well-form; that is, to only those sets that we have a definable predicate for. These "constructible sets" would be those "definable in ZF by expressions which quantify only over sets which have been previously defined" [10], and would close such a universe off to any undecidable sets. Not only was Gödel able to prove that this constructible universe was a consistent model of ZFC, but he was also able to demonstrate the consistency of CH within this model ($2^{\aleph_0} = \aleph_1$). Intuitively, since working mathematicians and scientists can only talk about and experience the natural and real numbers, it is not empirical for other numbers (or sets) to intrude on that universe; therefore, such a mathematical universe is capped at $\aleph_1$ **[EN 12].** As mathematician Thomas Jech summarizes: "Gödel constructed a model of ZFC, the constructible universe *L*, that satisfies CH. The model *L* is basically the *minimal possible collection of sets* that that satisfies the axioms of ZFC. Since CH is true in *L*, it follows that CH cannot be refuted in ZFC. In other words CH is consistent" [11].

However, it is clear that while this constructible universe contains all the known mathematics we currently experience, such a universe is in defeat of the original attempt to define the mathematical field free of *a posteriori* descriptions and predicates. This "constructivist" orientation subordinates all of what we can possibly know to the dominate language: "In its essence, constructivist thought is a logical grammer. Or, to be exact, it ensures that language prevails as the norm for what may be acceptably recognized as one-multiple amongst representations. The spontaneous philosophy of constructivist thought is radical nominalism... the [scientific] state is the master of language. Language-- or any comparable apparatus of recognition-- is the legal filter for groupings of presented multiples" [12].

One may quickly recognize in this constructivist orientation the promise of "scientific positivism"; a dogma that is still present in the sciences today as a mechanism to thwart off rival languages or epistemologies. Positivism claims science to have a privileged language for describing the world, and that at the end of scientific inquiry a well-formed language will be capable of describing all physical phenomena. This collusion between positivist dogma and constructivism does not escape Badiou's eye, and he is worth quoting at length:

"... one considers that the language of positive science is the unique and definitive 'well-made' language, and that it has to name the procedures of construction, as far as possible, in every domain of experience. Positivism considers that presentation is a multiple of factual multiples, whose marking is experimental; and that constructible liasons, grasped by the language of science, which is to say in a precise language, discern laws therein... one must confine oneself to controllable facts: the positivist matches up clues and testimonies, experiments and statistics, in order to guarantee belonging... *Under the injunction of constructivist thought, positivism devotes itself to the ill-rewarded but useful tasks of the systematic marking of presented multiples, and the measurable fine-tuning of languages. The positivist is a professional in the maintenance of apparatuses of discernment*" [13, *italics* mine]

Gödel's solution binds the universe to what can be empirically observed and tested ("discerned"), closing it off to any elements which may disrupt the language guiding these observations and tests. Positivism thereby proceeds through the construction of scientific theories rendered by a *pre-existing* and *dominate* language, but what this language deliberately casts aside to make its structure seemingly complete and consistent, through various unchecked assumptions and prejudices, is often overlooked. We will see a profound case of this in our study of the Einsteinian revolution.

[- Second Approach to the Continuum Problem: Cohen's Use of Forcing -]

What we must note is that while Gödel was able to show that CH is *consistent* with ZFC using the model of a constructible universe, he was still unable to prove CH conclusively. And this is where the *piece de resistance* of Badiou's *Being and Event* lies-- it is in reality for these well-made languages and constructible sets to be overrun and transformed by sets literally "forced" into a given situation. Such was the fruit of Paul Cohen's attempt to solve the Continuum Problem in 1966. Cohen developed a technique that he called "forcing", whereby he could create a model of ZFC where CH was deemed *false*. Jech recapitultes, "Cohen's accomplishment was that he found a method for constructing other models of ZFC. The idea is to start with a given model *M* (the *ground model*) and extend it by adjoining an object *G* [a "generic set"]... Cohen showed how to find (or imagine) the set *G* so that CH fails in *M[G]*. Thus CH is unprovable in ZFC…"[14]. The forcing of a generic set into a constructible universe invalidates CH, and $2^{\aleph_0} \neq \aleph_1$.

We mentioned that a direct outcome of the Axiom of Foundation is that *every structure has an undecidable impasse.* It is these undecidable sets that Badiou (after Cohen) terms "indiscernible" or "generic" sets; sets that are *not* constructible by the current language or its properties either because they were deliberately foreclosed from the situation, or because they have yet to be discovered **[EN 13].**

The derivation of the forcing method, forcing conditions, and subsequent transformations is notoriously complex and difficult **[EN 14].** However, we can detail the major properties and consequences. If we start with a given (constructible) situation composed of sets, the situation will naturally contain "discernible" sets, or sets that can be deemed true or false via the situation's well-made language **[EN 15]**. However, there will also exist indiscernible sets, which do *not* disclose their elements to the situation. "Thus for an inhabitant of *S*(*G*) [any situation with a generic set], there does not exist any intelligible formula in her universe which can be used to discern *G*." [15].

How these indiscernibles are "grasped" by those in the situation involves a theory of "forcing conditions": "... the elements of the base multiple chosen in the fundamental quasi-complete situation will be called *conditions* (for the indiscernible *G*)… certain groupings of conditions, conditions which are themselves conditioned *in the language of the situation*, will make it possible to think [an indiscernible multiple]... The idea is then that of seeing what happens if, by force, this indiscernible is 'added' or 'joined' to the situation... from a given situation, one can construct another situation by means of the 'addition' of an indiscernible multiple of the initial situation" [16]. The generic set is captured, not through some "mystical" language, but by using approximate local descriptions which show themselves as "true" or "false" as the generic set is incorporated into the situation and tested for its veracity amongst all the other sets and elements of the situation.

In this way, interested inhabitants of the situation can attempt to apprehend a generic set using their own language, and "force" it to be part of their situation. This procedure requires what Badiou calls an initial "nomination" or "declaration" of the generic set, followed by an "investigation" by *committed* groups or individuals, where they must assess, point-by-point, how inclusion of this generic set changes the languages and existences of their initial situation (this procedure the same as which occurs on the mathematical level when a generic set is forced in Cohen's method). Commitment and perseverance are important here, as the novel or disregarded nature of a generic set is likely to come up against the laws, rules, and dictates of the quasi-complete situation **[EN 16]**.

Thus is the method of scientific discovery *par excellence*-- incorporation of the unknown, tested for its veracity against every known piece of the current situation by committed individuals, typically against the contemporary tide. What we see in the procedure of forcing is an inherent ability, right at the foundation of mathematics, to inject novelty and true change into a given situation set, whether it be political, artistic, or scientific. It is, at its heart, the true *scientific* procedure-- an *egalitarian* procedure-- taking finite and incomplete information about something new or put aside in a world, and testing to see how it impacts the current state of affairs for *all*.

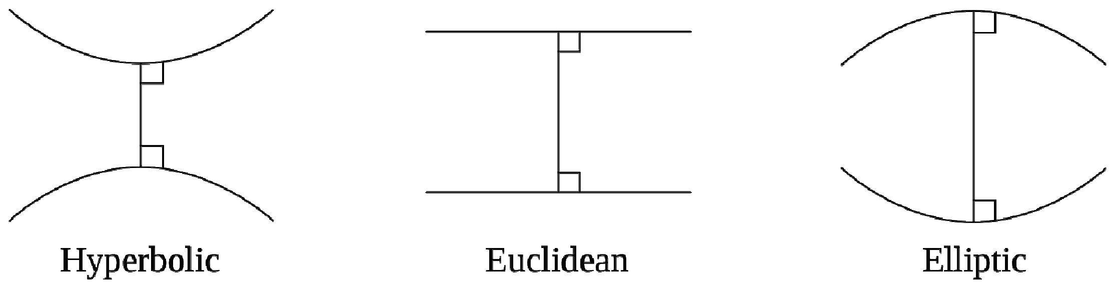
In going from Gödel to Cohen, we go from a closed and fully describable universe to one that is infinitely open to novelty and discovery; from one we may eternally hold, to one we must forever grasp at.

[- Case Study: Non-Euclidean Geometry and the Einsteinian Revolution -]

We will now demonstrate the importance of forcing through an historical example. Badiou and his many commentators utilize examples ranging from the political, to the artistic, to the mathematical. However, they hardly use any examples from the physical sciences. I hope that this brief exposition will inspire more to follow along this path **[EN 17]**.

Let us take the Newtonian universe as our situation. This is a situation that thought itself virtually consistent and complete: the universe consisted of point particles that could be fully characterized by their mass and velocity, and all abide by Newton's laws. The latent assumption of this entire system was deceivingly simple: the universe followed a Euclidean geometry.

Geometry came into being circa 300BC with the writing of Euclid's legendary *Elements*. Euclidian geometry follows from five basic postulates, with the controversy being in the fifth postulate, or the "Parallel Postulate" **[EN 18]**. It wasn't until the 18^{th} century (1,400 years!) that doubt grew over this postulate. Two "non" Euclidian geometries (what later became known as Lobachevskian/Hyperbolic and Riemannian/Elliptic geometry, respectively) were both put forward as replacements for the shaky fifth postulate. We shall have to pass over the details of what is, once again, a fascinating history of development, but these two geometries primarily result in *curved* spaces **[EN 19]:**



The very possibility of these non-Euclidian geometries posed a huge problem for how the physical universe's geometric foundations were perceived. As with the history of set-theory, the goal was always toward a singular, rigid, immovable foundation. Howard DeLong notes: "…society may develop customs to make it more unlikely that [an expert's] belief be questioned. We may then speak of a social prejudice. In this sense there was in Europe at the end of the 18th century a social prejudice consisting of the belief that Euclidean geometry was true... the intellectual climate was such that a questioning of this tradition met not so much with refutation as with derision" [17].

As we now know, it took committed individuals, such as Albert Einstein, to try and incorporate these suppressed geometries into our current scientific worldview. And even for Einstein, it had to start as a *thought* experiment prior to any possible empirical testing. DeLong notes, "... how about empirical observation? Is Riemannian geometry true of the world? Unfortunately, very few people in the 19th century took this question seriously. Riemann did... But Riemann died at 39... perhaps [he] would have preceded Einstein in the Theory of Relativity, for that theory proposes that a generalized version of Riemannian geometry is true of physical space" [18].

Einstein, however, took it very seriously. As Michael Freidman assesses: "... these mathematical and philosophical developments [concerning non-Euclidian geometries] formed the indispensable background to Einstein's formulation of the theory of relativity, and they were taken as such by Einstein himself... the lesson of the theory of relativity is not, as one might expect, that Euclidean geometry is a false description of physical space. It is rather, following [mathematician Henri] Poincare, that *there is no fact of the matter about the geometry of physical space: choosing one or another physical geometry is not forced upon us by any observable facts but rather depends on a prior convention or stipulation without which the question of physical geometry is simply undefined*" [19, *italics* mine].

It is clear that non-Euclidean geometry was an *undecidable proposition* and represented an indiscernible set of the Newtonian situation. In order for the Newtonian world to appear complete and consistent, a dogma was propagated to claim such geometries as absurd and practically useless. Through the forcing of non-Euclidian geometry into the Newtonian system, and the testing of its consequences point-by-point, the fabric of Nature itself was rewoven.

[- Conclusion -]

Through our exposition of set-theory and the CP approaches, we have shown how the very mathematics that we take for granted to rigidly ground our physical sciences is *itself* riddled with paradox and complexity. Of course, we may (naively) reject this history and its results, but even "those few mathematicians who still reject Cantor's theory must also reject large parts of classical mathematics… [set theory] provided a blow to the notion that our clear and distinct intuitions are criteria for truth" [20]. While this may permanently shatter our traditional visions of the physical sciences achieving some sort of unchanging and Archemedian view on the universe, we have also seen how this modern understanding of mathematics allows *within its very being* a way for us to apprehend and incorporate those theories we have unjustly placed aside and those great discoveries still yet to be had.

I propose that there should be a greater interest on the part of working scientists to address how mathematics directly impacts their theory choice. More importantly will be an assessment of how those theories we have discarded or prejudiced can still exert far-reaching effects on our current scientific principles. Badiou and his acolytes have already begun such historiographical cataloguing and description of these "inexistents" in other fields. This post-modern idea of *all* structures (whether scientific, political, artistic, etc.) containing congenital defects and impasses which allow for their eventual deconstruction and rebuilding is one Badiou has successfully been able to formalize and make accessible to us scientists in our very own mathematical language. I hope that this paper will motivate more scientists to read (and undoubtedly struggle with) Badiou's work and its consequences, as well as inspire them to interrogate their own fields and the place of science as a whole.

It is mathematics we must assert as the privileged language of our thought and experience. There is an endless reality out there to be discovered, with the mere inconvenience that the more we try to structure that reality, the more slips through our mortal fingers. As we began with a quote by Stephen Hawking on how he viewed the impact of these foundational issues, let us also end with his reflection:

"Some people will be very disappointed if there is not an ultimate theory that can be formulated as a finite number of principles. I used to belong to that camp, but I have changed my mind. I'm now glad that our search for understanding will never come to an end, and that we will always have the challenge of new discovery. Without it, we would stagnate." [21]

References []

[1] S.W. Hawking, "Gödel and The End of Physics". http://www.hawking.org.uk/godel-and-the-end-of-physics.html (2003)

[2] Alain Badiou, *Le Nombre et les nombres.* Paris: Seuil (1990). *trans.* Robin Mackay. *Number and Numbers.* Cambridge: Polity (2008). pp.1

[3] Alain Badiou, *Number and Numbers*, 82

[4] Kurt Gödel, "What is Cantor's Continuum Problem?" *Am. Math. Monthly*, 54, 515-25

[5] Mary Tiles, *The Philosophy of Set Theory: An Introduction to Cantor's Paradise.* Oxford: Blackwell (1989). Ch. 5

[6] Mary Tiles, *The Philosophy of Set Theory*, Ch. 6

[7] Howard DeLong, *A Profile of Mathematical Logic*. New York: Dover (2004 reprint, originally 1970). Ch.3, *esp* pp.128

[8] Peter Hallward, *Badiou: A Subject to Truth*. Minneapolis: University of Minnesota Press (2003). pp. 340

[9] Howard DeLong, *A Profile of Mathematical Logic*, 195

[10] Mary Tiles, *The Philosophy of Set Theory,* 176

[11] Thomas Jech, "What is Forcing?" Notices of the American Mathematical Society, Vol. 55, No. 6 (2008) pp. 693

[12] Alain Badiou, *L'Être et l'événement*. Paris: Seuil (1988). *trans*. Oliver Feltham. *Being and Event*. London: Continuum (2005). pp. 287

[13] Alain Badiou, *Being and Event,* 292

[14] Thomas Jech, "What is Forcing?", 693

[15] Alain Badiou, *Being and Event,* 387

[16] Alain Badiou, *Being and Event,* 357-358

[17] Howard DeLong, *A Profile of Mathematical Logic*, 38

[18] Howard DeLong, *A Profile of Mathematical Logic*, 53-54

[19] Michael Friedman, "Kuhn and Logical Empiricism" in *Contemporary Philosophy in Focus: Thomas Kuhn*, edited by Thomas Nickles. Cambridge University Press (2003). pp. 23-34

[20] Howard DeLong, *A Profile of Mathematical Logic*, 80-81

[21] S.W. Hawking, "Gödel and The End of Physics".

End Notes [EN]

[EN 0] To be consistent, I will be reproducing Badiou's demonstrations of the set theoretical literature, where applicable. The only exception being in the symbolism of the "generic" or "indiscernible" set: while Badiou uses ♀ for philosophical reasons, I have used *G* to follow the mathematical convention.

[EN 1] Before we move forward, for those who would like an excellent and more "accessible" review of the initial path we shall take from Cantorian to Non-Cantorian set-theory, I highly recommend the 1967 Scientific American article co-written by Paul Cohen himself. *See:* Paul Cohen and Reuben Hersh, "Non-Cantorian Set Theory". Scientific American, Vol. 217, No.6 (1967) pp.104-116

[EN 2] "...set theory distinguishes two possible relations between multiples. There is the originary relation, *belonging*, written ∈, which indicates that a multiple is counted as element in the presentation of another multiple"**.** So if A = [x,y,{z,w}], then x∈A, y∈A, and {z,w}∈A. "But there is also the relation of *inclusion*, written ⊂, which indicates that a multiple is a sub-multiple of another multiple... the writing $\beta \subset \alpha$, which reads β is included in α, or β is a subset of α, signifies that every multiple which belongs to β also belongs to α: $(\forall\gamma)[(\gamma \in \beta) \longrightarrow (\gamma \in \alpha)]$". So if B = [{x,y,z}, h], then [{x,y,z}]⊂B, [h]⊂B, and [{x,y,z},h]⊂B. Note that inclusion can be reduced to the primary relation of belonging. [Alain Badiou, *Being and Event*, 81]

[EN 3] Power set Axiom (or Subset Axiom): "If *a* is a set and F(*x*) is any well-formed expression in the language of ZF [axiomatic set theory] with a single free variable, then there is a set *b* whose elements are those elements of *a* for which F(*a*) is true. $\forall x\ \exists y\ \forall z\ [z \in y \leftrightarrow z \in x\ \&\ F(z)]$" [Mary Tiles, *The Philosophy of Set Theory*, 122]. Note that the power set is simply the group of inclusions or subsets of the original set, as defined in **[EN 2]**.

[EN 4] If A = ∅, then B = p(A) = [∅]. C = p(B) = [∅, {∅}]. D = p(C) = [∅, {∅}, {∅, {∅}}]. And so forth. [Mary Tiles, *The Philosophy of Set Theory*, 125 and 134]

[EN 5] For Cantor's construction of ω_0 and $\aleph_0$, see Mary Tiles, *The Philosophy of Set Theory*, 104-107 and/or Howard DeLong, *A Profile of Mathematical Logic*, 71-81. While [∅, {∅}, {∅, {∅}}], for example, can be given a finite size, or "cardinality" of 3 (as it has 3 elements), when it comes to infinite quantities we must use a new counting system. These are the infinite, or "transfinite", cardinals designated by the aleph ℵ. So ω_0 is said to have a cardinality of $\aleph_0$ elements (an "infinity" of elements). Mind here that an unintended consequence of Cantor's construction is that there may be "many" infinities; the modern study of which is the study of "large cardinals".

[EN 6] This derivation of the set of real numbers involves Cantor's celebrated "diagonal argument". *See*: Mary Tiles, *The Philosophy of Set Theory*, 107-111 and/or Howard DeLong, *A Profile of Mathematical Logic*, 76-81

[EN 7] "ZF" stands for "Zermelo-Fraenkel" set theory, while ZFC is ZF + the Axiom of Choice.

[EN 8] Consider a set that does not contain itself as "normal" (for example, the set of all circles is not itself a circle). Now consider a set that contains itself as "abnormal" (for example, the set of all non-circles is itself a non-circle). Finally, consider the set of *all* normal sets-- is this set normal or abnormal? It cannot be normal, or it would contain itself and be abnormal. It cannot be abnormal, or it would not contain itself and be normal. This is Russell's paradox. (Adapted from Howard DeLong, *A Profile of Mathematical Logic*, 81-82).

[EN 9] Axiom of Foundation: "Any non-void [non-empty] set posses as least one element whose intersection with the initial set is void; that is, an element whose elements are not elements of the initial set. One has $\beta \in \alpha$ but $\beta \cap \alpha = \emptyset$. Therefore, if $\gamma \in \beta$, we are sure that $\sim(\gamma \in \alpha)$. It is said that β founds α, or is on the edge of the void [∅] in α" [Alain Badiou, *Being and Event*, 500]. Formally, $\forall x[\sim(x = \emptyset) \longrightarrow \exists y\ (y \in x\ \&\ \forall z\ (z \in x \longrightarrow \sim(z \in y)))]$ [Mary Tiles, *The Philosophy of Set Theory*, 122]

[EN 10] Badiou uses the term "situation" to stand in for any circumscribed group of infinite sets defined by a finite group of properties (or predicates). He simply defines a situation as "any consistent presented multiplicity, thus: a multiple [a set of sets], and a regime of count-as-one [a set of properties], or structure". Every situation is structured in a constructible fashion (as described later). Badiou is thus able to give concepts such as "political situation", "artistic situation", or "scientific situation" a rigorous set-theoretically based structure. The wager, again, of his work is that as paradoxes are part of the general set-theoretical edifice, so will they be of these situations. At these points of paradox, or stress, every situation is open to the forcing of "generic" sets that can overrun and transform that situation (again, described later). [Alain Badiou, *Being and Event*, 522 and Meditation One]

[EN 11] On what a "predicate" is: "The central notion to be explained is that of a propositional function (or *predicate*, as it will also be called). Let there be fixed some nonempty *domain of discourse*, that is, some set of objects which our logic will be about. Examples may include the set of physical objects, the set of living animals, or the set of natural numbers. The members of the domain of discourse will be called *individuals*. An *n-place propositional function* (or *n-place predicate*) is a function of *n* individual variables, where the domain of definition is the domain of discourse and the domain of values is a set of propositions. Hence, when each variable in a propositional function (predicate) has assigned to it an individual, the result is a proposition." [Howard DeLong, *A Profile of Mathematical Logic*, 111]. The notion of a predicate allows one to "separate" out the objects of our experience, as sets, *through the use of language*. These predicates may then be marked as "true" or "false" in their consistency with the rest of the predicates of the system. The crucial finding of Gödel's First Incompleteness Theorem is that there will *always* exist a predicate "undecidable" within the system—it cannot be shown to be either true or false. This undecidable set leaves the system either incomplete or inconsistent.

[EN 12] It is goal of Badiou's *Number and Numbers* to show this to be patently *false*, and the whole second half of the book is dedicated to expounding John Conway's theory of "surreal numbers", which suggests whole universes of numbers beyond what us scientists and mathematicians experience in practice. (*See also*: John Conway, *On Numbers and Games*. London: Academic Press (1976)). As one can imagine, the existence of such numbers would invalidate the Continuum Hypothesis, and be a support of Paul Cohen's Non-Cantorian set-theory (to be described).

[EN 13] "A part of the situation is indiscernible if no statement of the language of the situation separates it or discerns it. Or: a part is indiscernible if it does not fall under an encyclopaedic determinant". Formally, for a given situation S, and indiscernible set G: $\sim(G \in S)$, but $G \in S(G)$ if forced into S. [Alain Badiou, *Being and Event*, 386 and 512]

[EN 14] While Badiou provides a faithful account of the generic and forcing within his philosophical system (see *Being and Event*, Meditations Thirty-One, Thirty-Three, Thirty-Four, and Thirty-Six), the more mathematically inclined may want to reference the original source. *See*: Paul J. Cohen, *Set Theory and the Continuum Hypothesis*. New York: W.A. Benjamin (1966). For scientists and mathematicians, there is also Timothy Chow's brief beginner's guide to forcing. *See*: Timothy Y. Chow, "A beginner's guide to forcing" in *Communicating Mathematics*. Contemp. Math, Vol. 479 (2009), pp. 25–40 (also, hosted by the author at http://math.mit.edu/~tchow/forcing.pdf)

[EN 15] "A set δ is discernible *for an inhabitant of S* (the fundamental quasi-complete situation) if there exists an explicit property of the language of the situation which names it completely. In other words, an explicit formula $\lambda(\alpha)$ must exist, which is comprehensible for an inhabitant of S, such that 'belong to δ' and 'have the property expressed by $\lambda(\alpha)$ coincide: $\alpha \in \delta \leftrightarrow \lambda(\alpha)$" [Alain Badiou, *Being and Event*, 367].

[EN 16] A common political example Badiou likes to use is that of the proletarian under bourgeois rule. While the proletarian physically exist, they do not exist under the economic and political language dominated by the upper class. The proletarian therefore forms an "indiscernible" set of this economic-political situation, and they must revolt to "force" their universal egalitarian rights, testing their validity along the way. The same may be said of various civil rights movements.

[EN 17] Again, due to space limitations, we have to make this study extremely brief. However, it should suffice to demonstrate our argument. I hope to perform a more thorough and rigorous study of this topic in the future.

[EN 18] The most widely cited version of this postulate is John Playfair's: "through a given point, only one parallel can be drawn to a given straight line" [Howard DeLong, *A Profile of Mathematical Logic*, 42]. This would give us a traditional linear Euclidian geometry-- two independent lines must be parallel.

[EN 19] An intuitive way to think of the differences between each geometry is that while the sum of the angles of a triangle must always equal 180° in Euclidian geometry, they will be *less* than 180° in Hyperbolic geometry, and *greater* than 180° in Elliptic geometry.