Unification of Nuclear Structure Theory Is Possible

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Abstract

The impossibility of achieving a unified theory of nuclear structure has been the conventional wisdom in nuclear physics since the 1960s. However, already in 1937 Eugene Wigner indicated a way forward in theoretical work that eventually led to a Nobel Prize, but not directly to unification. Specifically, he showed that the symmetries of the Schrodinger equation have an intrinsic face-centered-cubic (FCC) geometry. Those symmetries provide for a fully quantum mechanical unification of the diverse models of nuclear structure theory, as indicated by the following facts: (i) The FCC lattice reproduces the properties of the liquid-drop model due to short-range nucleon-nucleon interactions (constant core density, saturation of binding energies, nuclear radii dependent on the number of nucleons, vibrational states, etc.). (ii) There is an inherent tetrahedral subgrouping of nucleons in the close-packed lattice (producing configurations of alpha clusters identical to those in the cluster models). And, most importantly, (iii) all of the quantum n-shells, and j- and m-subshells of the independent-particle model are reproduced as spherical, cylindrical and conical substructures within the FCC lattice – with, moreover, proton and neutron occupancies in each shell and subshell identical to those known from the shell model. These facts were established in the 1970s and 1980s, but the "impossibility of unification" had already achieved the status of dogma by the 1960s. Here, I present the case for viewing the lattice model as a unification of traditional nuclear structure theory – an unambiguous example of how declarations of the "impossibility" of progress can impede progress.

I. Introduction

There is no greater obstacle to progress than a belief that progress is impossible. Unsolved puzzles and indecipherable mysteries can be found in any academic field in any era – and textbook authors inevitably treat the unsolved problems as beyond the powers of "modern" science. Today is no exception. For example, the advances in quantum chromodynamics (QCD) have brought theoretical coherency to the world of particle physics, but, despite remarkable precision of the experimental data, the masses of the 200+ elementary particles remain a puzzle beyond the scope of QCD. As Feynman (1985) has commented: "There remains one especially unsatisfactory feature: the observed masses of the particles. There is no theory that adequately explains these numbers… This is a very serious problem" (p. 152). But it is an unsolved problem primarily because it is so rarely addressed (e.g., MacGregor, 2007; Palazzi, 2003; 2004).

At the level of nuclear structure, the textbooks state that the nucleus is such a complex many-body problem that inherently-incomplete, mutually-contradictory "models" are necessary to explain the diverse properties of nuclei, and that a truly unified theory is not possible. That view has been widely espoused since the late 1950s, when the vastly different ***gaseous-phase*** independent-particle model (IPM), ***liquid-phase*** liquid-drop model (LDM) and molecule-like ***solid-phase*** cluster (~alpha-particle) models were all found to have realms of quantitative applicability, based upon assumptions about nucleons and the nuclear force that explicitly contradict the assumptions in other models. They were all deemed to be "correct" within a specific range of applications, but together did not add up to a coherent, unified theory.

Already in the 1930s, however, Wigner (1937) demonstrated the geometrical simplicity of the quantum mechanics of nucleon states. That geometry, i.e., a face-centered-cubic (FCC) lattice, forms the basis of a model that contains within it the liquid-drop approach (a short-range nuclear force and locally-interacting, space-occupying nucleons), the shell-model approach (independent nucleon states forming shells and subshells) and the cluster-model approach (non-liquid and non-gaseous configurations of alpha particles). For historical reasons (concerning the reformulation of the Schrodinger equation in light of spin-orbit coupling), Wigner's unifying idea was not pursued in the 1940s, and the gaseous-phase shell model became the dominant paradigm in nuclear structure theory. Since then, however, a small group of physicists have continued to develop Wigner's original insight and have demonstrated the viability of the unification of nuclear theory within the framework of an FCC lattice of nucleons.

II. A Brief History of Nuclear Structure Theory

The early era of nuclear modeling saw the introduction of the LDM (1930s) to account for many of the outstanding properties of the nucleus: nuclear sizes, nuclear binding energies and, most notably, fission phenomena. During the 1930s and 1940s, the cluster models were also developed to account for the unusual stability and abundance of the 4n-nuclei (He^4, C^{12}, O^{16}, …, Ca^{40}) and to explain the fact that alpha particles were emitted from certain large nuclei. In 1949, the shell model was introduced. Unlike both the LDM and cluster models, the shell model: (i) was based on the Schrodinger equation, (ii) was formally related to the quantum mechanics of atomic (electron) structure, and (iii) was therefore welcomed by theorists as a fundamental ***theory***, as distinct from the various analogies with macroscopic objects that had previously been developed as nuclear "models." As a consequence, since the 1950s the shell model has been the central paradigm in nuclear structure theory, but all three approaches have well-established, quantitative uses that the other models cannot mimic. In spite of the reality of numerical conflicts among these models, the gaseous-phase, liquid-phase and solid-phase cluster models have typically occupied sequential chapters in nuclear physics textbooks since the 1960s.

As if nuclear structure theory were not complex enough, subsequent to the rapid advances in computing in the 1970s and 1980s, algorithms using nucleon lattices were developed for simulation studies of heavy-ion multifragmentation. The lattice models were found to be particularly useful at relatively high-energies, where the established nuclear structure models were inapplicable. Contrary to the expectations of many, the "experimental theory" of computer simulations produced results "with perplexing accuracy, despite the dearth of nuclear physics content" of the lattice models (Moretto & Wozniak, 1993, p. 450). Primarily on the strength of the multifragmentation simulation results, the lattice models (e.g., Bauer, 1988; Campi, 1988; Chao & Chung, 1991; DasGupta et al., 1996) and related "molecular dynamics" simulation techniques joined a long list of useful, but inherently-incomplete approaches to the nuclear many-body problem. By the 1990s, more than 30 variants of these liquid-phase, gaseous-phase, cluster and lattice approaches to nuclear structure were in use (Greiner & Maruhn, 1996). Despite remarkable developments in nuclear technology and nuclear experimentation during the previous six decades, nuclear structure theory – ***unlike all other branches of quantum theory at the atomic, molecular and solid-state levels*** – failed to evolve toward unification. As a consequence, the use of inherently incompatible models was widely considered to be an unavoidable, if temporary, strategy in the study of nuclear physics.

By the close of the 20th Century, the consensus view was that nuclear structure theory already had more than enough "models" and the time had come for harnessing computer power for rigorous ***ab initio*** calculations. Although hard- and software progress has of course been significant, even today the prospects for rigorously computing the structure of medium-sized nuclei, much less 235-nucleon systems, remain bleak. The complexities of nuclear structure theory are of course born of the fact that nuclei contain too many constituents for exact, analytical solutions, but too few constituents for rigorous stochastic approximations. In between those two extremes, “models” of varying realism and reliability can be usefully employed – and, if theoretically unsatisfying, few nuclear physicists believe that nuclear theory is in a state of crisis. On the contrary, nuclear structure theory is often said to be a “closed chapter” in microphysics. Today, most research on nuclear structure involves primarily short-lived exotic states, nuclear physics is no longer the field with the largest number of PhD students worldwide, and indeed monographs from the 1950s and 1960s are reprinted ***in unaltered form*** and used as college textbooks (Blatt & Weisskopf, 1954/1991; Landau & Smorodinsky, 1959/1993; Bohr & Mottelson, 1969/1998). In other words, progress in nuclear structure theory has come to a halt.

Nonetheless, genuine puzzles remain. Most notably, advances in experimental and computational nuclear physics have not led to an understanding of the nuclear force and the fundamental nuclear “equation of state” is still unknown. Even the phase of nuclear matter remains an open question – the problem often being stated in terms of (i) the Coester band (the discrepancy between estimates of the nuclear density and the nuclear binding energy), (ii) the length of the mean-free-path of nucleons “orbiting” in the nuclear interior (long for the shell model, short for the LDM), or (iii) the dimensions of the “realistic” versus the “effective” nuclear force interaction between nucleons in bound nuclei (with again the various nuclear models demanding radically different assumptions).

Meanwhile, with no resolution of the dilemma of multiple models in nuclear theory on the horizon, the vast majority of nuclear physicists have in fact moved on to the experimentally more difficult, but theoretically “cleaner” issues of QCD – in the hope that answers to questions concerning the nuclear force and the quark substructure of the nucleon will eventually lead back to clarification of nuclear structure. Progress in particle physics has consequently been stupendous, but no consensus on fundamental issues in nuclear structure theory has yet emerged.

III. The Lattice Model

The history of the unification of nuclear structure theory within a lattice of nucleons begins with a paper by Eugene Wigner (1937). There, and in subsequent theoretical papers on the “symmetries of the nuclear Hamiltonian,” he outlined the basic quantum mechanical properties of the nucleus (Figure 1) – work that eventually led to a Nobel Prize explicitly for his contributions to an understanding of nuclear quantum mechanics.

Wigner’s quantal formalism immediately became the basic theoretical tool for describing nuclear states. In the late 1940s, that description was developed into the shell model with reformulation of the nuclear energy-levels based on the idea that there is a coupling of orbital and intrinsic angular momentum. So-called spin-orbit coupling meant that each nucleon had a total angular momentum, ***j***, which was an observable property of all odd-Z and/or odd-N nuclei. The agreement with experimental data was remarkably accurate, and the 1961 Nobel Prize went to Wigner (50%) for establishing the IPM and Goeppert-Mayer and Jensen (25% each) for the shell model variation on the IPM.

Eventually, the shell model became predominant, but the debate concerning the various nuclear models was acrimonious in the early 1950s when the stark differences between the liquid-phase LDM (essential for all work in fission) and the equally-successful gaseous-phase shell model approaches became apparent. On the one hand, the successes of the IPM in predicting nuclear spins strongly indicated the “independent-particle” nature of the nucleus, but explanations of nuclear radii, densities, vibrations and the release of energy in nuclear fission required the LDM, which is explicitly a “collective” liquid-phase model. Meanwhile, the process of alpha-particle emission from certain large nuclei and the ability of the cluster models to predict the electron form-factors and low-lying excited states of the small 4n-nuclei indicated the reality of alpha-particle clustering in the nuclear interior and on the nuclear surface of many, perhaps all, nuclei. As a consequence, the mutually-exclusive successes of the various nuclear models were implicitly elevated to the status of ***yet-another unavoidable “paradox” of quantum physics***, and several generations of students in nuclear physics have learned to accept the counter-intuitive disunity of nuclear structure theory as the final answer.

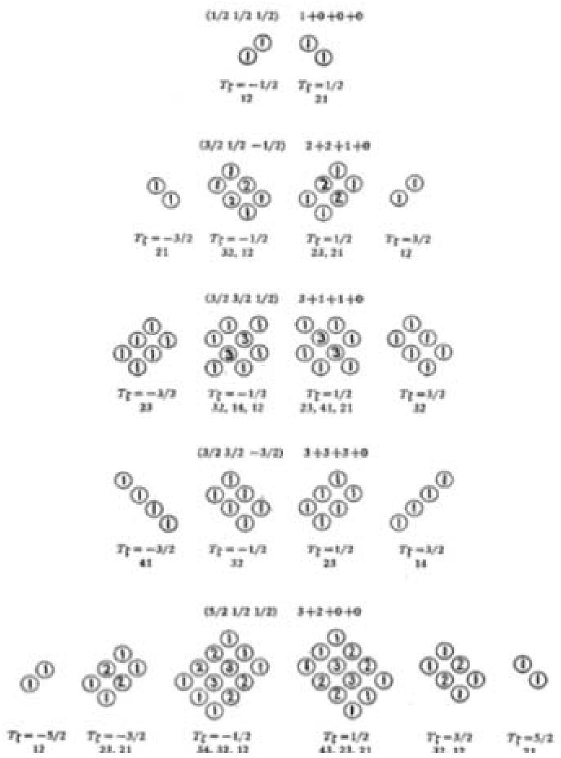

Figure 1: The 2D symmetries of the quantum numbers for He^4, O^{16}, Si^{28}, Ne^{20} and Ca^{40}, as depicted by Wigner (1937, his Figure 1).

Although it is today a matter of “common sense” that resolution of the paradoxes of nuclear structure theory is impossible, a return to the geometric symmetries discovered by Wigner shows a straight-forward unification of the nuclear models. Specifically, Wigner’s 1937 illustration of nuclear quantal symmetries (Figure 1) demonstrated that, when each nucleon is depicted as a space-occupying particle with nearest-neighbors in the same plane and in planes above and below, the symmetries of the nuclear Hamiltonian as a whole are those of a 3D FCC lattice (as noted by Wigner himself (1937, p. 108). In other words, the “close-packing” of nucleons – as assumed in the liquid-drop and lattice models – produces the exact same quantal symmetries that the gaseous-phase shell model is famous for. There is consequently no paradox in describing nuclei as consisting of “independent” nucleons in a “collective” regime, but the nucleus is clearly ***not*** a gas of non-interacting nucleons.

A. The face-centered-cubic (FCC) symmetries

The “convenient fiction” that underlies the modern shell model is that nucleons are “point” particles and, to a first approximation, do not interact with one another locally within the nucleus. To the contrary, however, individual nucleons are known experimentally to be space-occupying particles (RMS radius ~1 fm) (Table 1, Figure 2), whose localization in the nucleus can be expressed as a Gaussian probability function, consistent with the uncertainty principle (Lezuo, 1974). As such, within the framework of the LDM, cluster and lattice models, all of the non-classical aspects of quantum mechanics are confined to the description of the individual nucleon, whereas the properties of multi-nucleon nuclei can be calculated simply as the summation of the features of nucleons. Provided only that the realistic dimensions of the nucleons, as shown in Figure 2, are assumed, a dense liquid or dynamic lattice model of the nucleus is inevitable.

The FCC lattice model, in particular, has the macroscopic properties of a dense liquid-drop (with a solid-phase interior and superfluid surface). Shell structure and internal tetrahedral “clustering” of nucleons within the close-packed lattice, as described below, but the individual nucleons are fundamentally quantum mechanical. The principal attraction of the FCC nuclear model lies in Wigner’s discovery that the entire systematics of nucleon quantum numbers (known today in the form of the IPM) are uniquely reproduced in an antiferromagnetic FCC lattice of nucleons with alternating isospin layers (Figure 4). This is precisely the same configuration of nucleons that has been shown to be the lowest energy condensate of nuclear matter (N=Z), probably present in the crust of neutron stars (Canuto & Chitre, 1974).

Electric and magnetic rms radii of nucleons

Particle	r^{e}_{rms}	r^{m}_{rms}
Proton	0.895 ± 0.018 fm	1.086 ± 0.012 fm
Neutron	-0.113 ± 0.003 fm	0.873 ± 0.015 fm

I. Sick, *Progress in Particle and Nuclear Physics* 55, 440, 2005.

Table 1: A summary of electromagnetic measures of the nucleons.

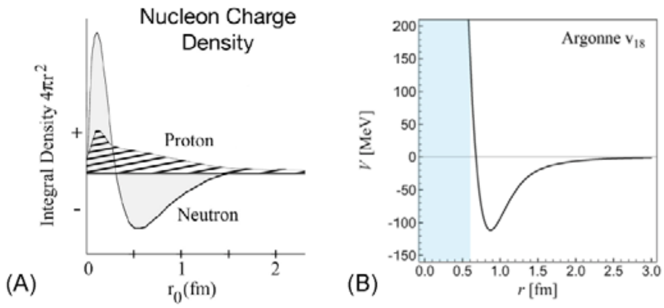

Figure 2: The dimensions of the nucleon (A) and the nuclear force (B), indicating effects up to ~2.0 fm from the center of the nucleon.

The well-known successes of the IPM itself are based on a quantum mechanical description of all possible nucleon states, as given by the Schrodinger equation (Eq. 1, Table 2):

$$\Psi_{n,j\,(l+s),m,i} = R_{n,j\,(l+s),i}\,(r)\; Y_{m,j\,(l+s),i}\,(\theta, \phi) \qquad \text{Eq. 1}$$

By providing a rigorous foundation for describing individual nucleons, the IPM made it possible to calculate nuclear states as the summation of the properties of its “independent” nucleons. Those predictions were and still are important theoretical successes, and played a significant role in the establishment of the IPM in the early 1950s. Despite the counter-intuitive (and, circa 1950, vehemently disputed) “gaseous” nuclear phase-state implied by the IPM, its universally-acknowledged strength lay in the fact that each “independent” nucleon in the model had a unique set of quantum numbers (***n, j, m, l, s, i***), as specified in the nuclear version of the Schrodinger equation. The main short-coming of the IPM was the assumption of “point” nucleons, and the related neglect of local particle interactions, but it is specifically this assumption that the FCC model rejects, while retaining the IPM description of individual nucleon states. Of course “local interactions,” i.e., short-range (2-3 fm) effects, are the essence of the LDM – which works strictly on the basis of a realistic nuclear force, as known from nucleon-nucleon scattering experiments, and with no long-range “effective” nuclear force whatsoever. The FCC model shares this property of nucleons with the LDM.

It should be noted that Eq. 1 differs from the Schrodinger equation used in atomic physics primarily in the addition of isospin ***(i)***, indicating two varieties of nucleon, and the specification of the total angular momentum ***(j)*** as the sum of orbital ***(l)*** and intrinsic ***(s)*** angular momentum. Regardless of the notoriously complex spatial topology of the spherical harmonics, Y(θ, ϕ), the state of each nucleon in the IPM is defined by its unique set of quantum numbers, the sum total of which provides, in principle, a complete description of the energetic state of the nucleus as a whole. The experimental reality of unique quantal states for the nucleons in bound nuclei has been verified countless times since the 1930s (most importantly, measurements of nuclear angular momenta and magnetic moments) and has made the IPM the central paradigm of nuclear theory.

Table 2: The fundamental quantization of nucleon states and the occupancy of shells.

Energy ($E/h\omega_o$)	N-shell ($(n_x+n_y+n_z-3)/2$)	Wave-functions (n_x n_y n_z)	No. of Distinct Wave-functions	Spin Degeneracy	Isospin Degeneracy	Total Occupancy
15/2	6	(13 1 1)(1 13 1)(1 1 13) (11 3 1)(11 1 3)(3 11 1) (1 11 3)(3 1 11)(1 3 11) (951)(915)(591) (195)(519)(159) (933)(393)(339) (771)(717)(177) (753)(735)(357) (537)(573)(375) (555)	28	56	112	336
13/2	5	(11 1 1)(1 11 1)(1 1 11) (931)(913)(391) (193)(319)(139) (751)(715)(571) (175)(517)(157) (733)(373)(337) (553)(535)(355)	21	42	84	224
11/2	4	(911)(191)(119) (731)(713)(371) (173)(137)(317) (551)(515)(155) (533)(353)(335)	15	30	60	140
9/2	3	(711)(171)(117) (531)(513)(351) (153)(315)(315) (333)	10	20	40	80
7/2	2	(511)(151)(115) (331)(313)(133)	6	12	24	40
5/2	1	(311)(131)(113)	3	6	12	16
3/2	0	(111)	1	2	4	4

The range of values that the quantum numbers in Eq. 1 can take is known to be:

$\boldsymbol{n}$ = 0, 1, 2, … Eq. 2
$\boldsymbol{j}$ = 1/2, 3/2, 5/2, …, (2$\boldsymbol{n}$+1)/2 Eq. 3
$\boldsymbol{m}$ = -$\boldsymbol{j}$, …, -5/2, -3/2, -1/2, 1/2, 3/2, 5/2, …, $\boldsymbol{j}$ Eq. 4
$\boldsymbol{s}$ = 1/2, -1/2 Eq. 5
$\boldsymbol{i}$ = 1/2, -1/2 Eq. 6

Together with the Schrodinger equation itself, Eqs. 2~6 are essentially a concise statement of the quantum mechanics of the IPM, from which the “magic” numbers of the shell model can be obtained by manipulations of the nuclear potential well. From the point of view of the unification of nuclear structure models, what is of interest about the conventional IPM (Table 2) is that the standing-waves of the wave-functions (n_x, n_y, n_z) specify the location of distinct nodes – and are found to define (one quadrant of) an FCC lattice. Although neither Wigner nor the inventors of quantum mechanics had a lattice model of the nucleus in mind while deciphering the symmetries of nuclear states, the Schrodinger equation that defines “quantum space” simultaneously provides nucleon positions in the coordinate space of an FCC lattice.

The identity between quantum space and coordinate space can be stated either as the definition of FCC lattice sites for each nucleon in terms of its quantum numbers (Eqs. 7-9), or, vice versa, the unique Cartesian coordinates for each nucleon can be used to define its quantal characteristics (Eqs. 10-14):

$$x = |2\boldsymbol{m}|(-1)(\boldsymbol{m}+1/2) \qquad \text{Eq. 7}$$
$$y = (2\boldsymbol{j}+1-|x|)\text{^}(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{m}+1/2) \qquad \text{Eq. 8}$$
$$z = (2\boldsymbol{n}+3-|x|-|y|)\text{^}(\boldsymbol{i}+\boldsymbol{n}-\boldsymbol{j}-1) \qquad \text{Eq. 9}$$
$$\boldsymbol{n} = (|x| + |y| + |z| - 3) / 2 \qquad \text{Eq. 10}$$
$$\boldsymbol{j} = (|x| + |y| -1) / 2 \qquad \text{Eq. 11}$$
$$\boldsymbol{m} = \boldsymbol{s} * |x| / 2 \qquad \text{Eq. 12}$$
$$\boldsymbol{s} = (-1)\text{^}(x-1) / 2 \qquad \text{Eq. 13}$$
$$\boldsymbol{i} = (-1)\text{^}(z-1) / 2 \qquad \text{Eq. 14}$$

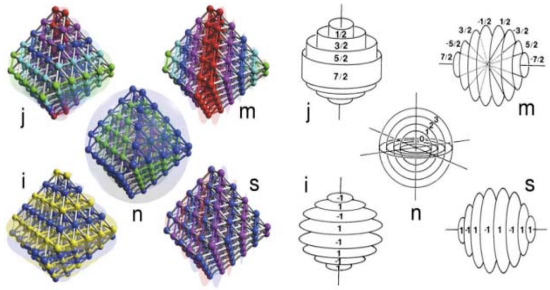

Figure 3: The FCC symmetries of nucleon quantum numbers.

Either way, the essential point is that the known quantum numbers and the occupancy of protons and neutrons in the ***n***-shells and ***j***- and ***m***-subshells are identical in both descriptions. As illustrated in Figure 3, the abstract symmetries of the Schrodinger equation have related symmetries in coordinate space: ***n***-, ***j***- and ***m***-shells have spherical, cylindrical and conical symmetries, respectively, while ***s***- and ***i***-values produce orthogonal layering. Examination of the symmetries of the structures in Figure 4 in relation to their Cartesian coordinates will show the validity of these equations for the unit structure of the FCC lattice model. More complex FCC structures are easily examined using software designed for that purpose (Cook et al., 1999). The implications of this precise and mathematically unambiguous isomorphism have been elaborated as the FCC nuclear lattice model in publications by a dozen authors over the past three decades, and recently summarized in a monograph (Cook, 2006).

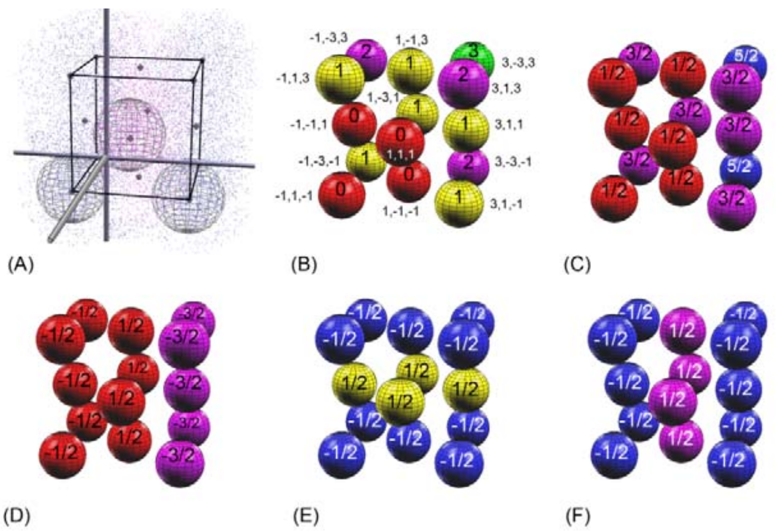

Figure 4: Six depictions of the 14-nucleon "unit structure" of the FCC lattice. The unit structure corresponds to a highly unstable isotope of Beryllium, and is shown here only to illustrate the precise geometry of quantum numbers in the lattice. (A) shows the Gaussian "probability clouds" of the 14 nucleons, with the 90% probability wire-spheres illustrating the known dimensions of nucleon size (r=0.86 fm) and nuclear density (0.17 nucleons/fm^3). (B)~(F) illustrate the assignment of quantum numbers depending solely on nucleon lattice coordinates. (B) Principal quantum number ***"n"*** (red = 0, yellow = 1, purple = 2, green = 3). (C) Total angular momentum number ***"j"*** (= | ***l+s*** |) (red = 1/2, purple = 3/2, blue = 5/2). (D) Azimuthal quantum number ***"m"*** (red = | 1/2 |, purple = | 3/2 |). (E) Isospin quantum number, ***"i"***, (yellow = 1/2, blue = -1/2). (F) Spin quantum number ***"s"*** (purple = 1/2, blue = -1/2).

Clearly, the significance of Eqs. 1~14 lies in the fact that, if we know the IPM (~shell model) structure of a nucleus, then we also know its FCC lattice model structure, and vice versa. The only structural uncertainties in ***both*** models come from the fact that only the quantal characteristics of the last-odd proton and/or last-odd neutron are known unambiguously from experiment. Even-Z and even-N nuclei are assumed to have paired valence nucleons, differing only in spin, and the core nucleons are assumed to have the same IPM characteristics as known from smaller (odd-Z and/or odd-N) nuclei. Both of these latter assumptions are generally well-justified, but there are in fact many known cases of intruder states and configuration-mixing in which the "default" IPM nucleon build-up sequence is not followed.

Stated conversely, the ***difference*** between the IPM and the FCC lattice model lies primarily in their implications concerning the local substructure within the nucleus. The IPM maintains that substructure is a direct consequence of energy gaps in a long-range, "effective" nuclear potential-well, whereas the lattice model views the same configuration of quantum states as a "dense liquid-drop" held together by a realistic, short-range nuclear force, with substructure determined by local nucleon-nucleon interactions. In this respect, the lattice model has properties similar to both the IPM and the LDM, but the lattice has additional substructure not found in either a liquid-drop or a nucleon "gas" of independent particles. That substructure allows for predictions concerning many nuclear properties (Cook, 2006). Notable divergences with IPM and LDM predictions include the prediction of the ***impossibility*** of stable or long-lived super-heavy nuclei (Z>112) and the prediction of ***asymmetrical*** fragments produced by the thermal fission of the actinides (Cook, 1999; Cook & Dallacasa, 2009).

B. The Shells and Alpha Clusters in the FCC Lattice

The consecutive ***n***-shells implied by the FCC lattice (built from a central tetrahedron) are identical to those of the isotropic harmonic oscillator (Figure 5). These correspond to the doubly magic nuclei for ***n***=0, 1 and 2, whereas the higher magic numbers in both the shell model and the FCC model require consideration of ***j***-subshells (Figure 6).

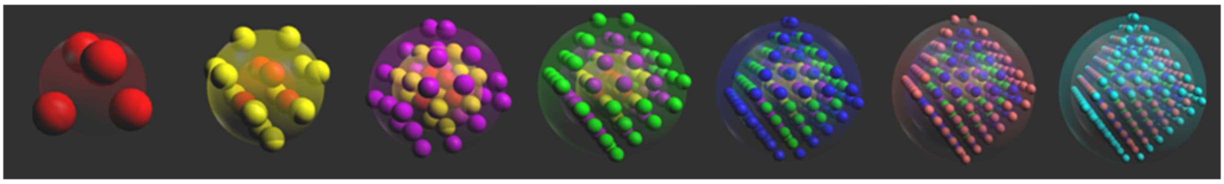

Figure 5: The tri-axially symmetrical, closed structures corresponding to the first 7 ***n***-shells of the harmonic oscillator: He^4, O^{16}, Ca^{40}, Zr^{80}, Yt^{140}, Xx^{224}, Xx^{336}.

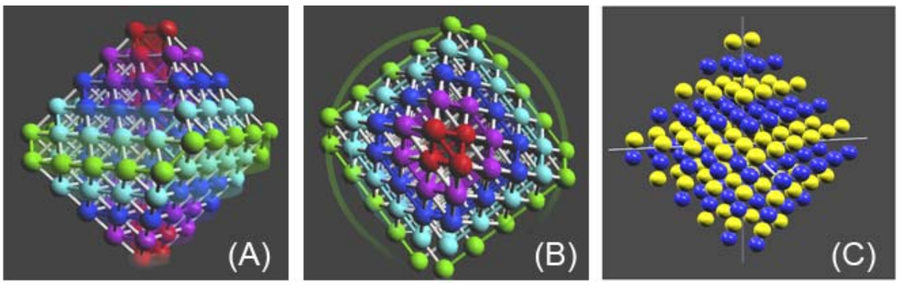

Figure 6: An example of the closed j-subshells within the closed n=4 nucleus. (A) Note the increasing occupancy of the subshells closer to the nuclear equator. The j=1/2 nucleons are red, 3/2 purple, 5/2 blue, 7/2 turquoise and 9/2 green. (B) Looking down the nuclear spin axis, the dependence of j on the nucleon's distance from the spin axis is apparent. (C) Alternating isospin layers mean that half of the nucleons in each j-subshell are protons, half neutrons.

That is, the closure of "magic" shells when N>Z entails the filling of a proton ***n***-shell and the next neutron ***j***-subshell. As a consequence of (i) influences of proton numbers on the filling of neutron shells (and vice versa), and (ii) the configuration-mixing of ***j***-subshells, the identification of "magicness" is empirically complex, but the symmetries of the quantum numbers in the Schrodinger equation constitute the unambiguous foundation for all theoretical manipulations. The isomorphism between the IPM and FCC lattice means that all of the IPM predictions of nuclear spin states, parities and magnetic moments are also found in the lattice model.

Finally, although the alpha cluster model remains a minority concern within nuclear structure theory, its successes are not easily interpreted within the framework of either a liquid-phase LDM or a gaseous-phase IPM, but find a surprisingly simple explanation within the FCC lattice model. Figure 7 illustrates how the FCC lattice contains inherent tetrahedral grouping of nucleons within the lattice and reproduces the symmetries of one of the clearest successes of the cluster models in explaining the electron form-factor and excited states of Ca^{40}.

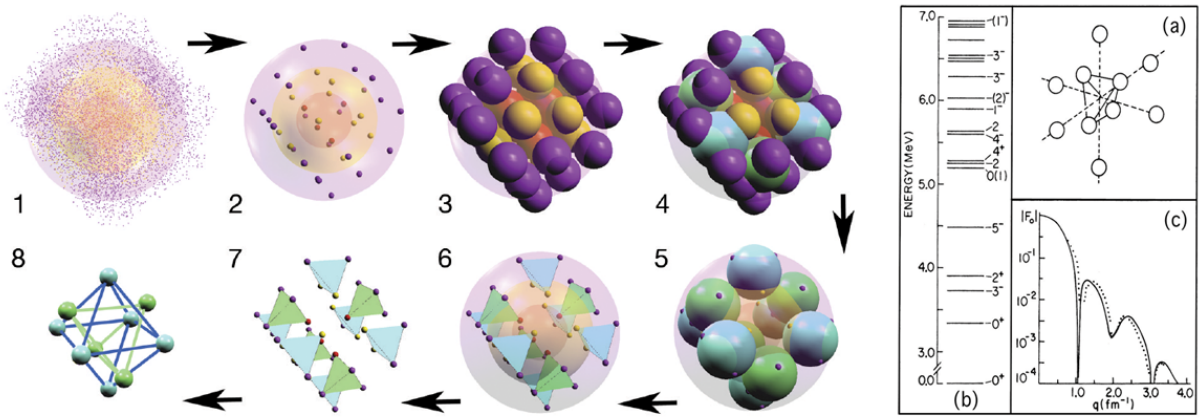

Figure 7: On the left, are shown 8 depictions of the Ca^{40} nucleus in the lattice model: (1) all nucleons depicted as probability clouds, (2) nucleons depicted as point particles in three distinct shells, (3) nucleons with realistic dimensions, (4) nucleons grouped into alpha clusters, (5) nucleons reduced in size to emphasize the cluster structure, (6) alphas depicted as tetrahedral within n-shells, (7) alphas only, (8) the geometry of the alphas. On the right is shown the same alpha geometry (e.g., Hauge et al., 1971) as in the FCC lattice (a), and the successful predictions of that geometry concerning excited states (b) and the electron form-factor for Ca^{40} (c).

IV. Conclusion

As paradoxical as it may first seem to be, the FCC lattice model exhibits properties that are normally attributed to the gaseous, liquid and cluster models of traditional nuclear structure theory. Most outstanding are: (i) the precise reproduction of the quantal symmetries of the IPM – leading to spin/parity characteristics of nuclei that are identical to the IPM, (ii) the LDM-like nuclear interior (nearest-neighbor, short-range nucleon-nucleon interactions) – leading to nuclear binding energy, density and radial measures similar to the LDM, and (iii) the alpha-particle subgrouping of nucleons within the lattice – leading to alpha clusters on the nuclear surface and throughout the nuclear interior. As reviewed above, the complete avoidance of an "effective" nuclear force, as employed in the shell model, is a major theoretical attraction.

The identity between the IPM and lattice model, described briefly above and in more detail elsewhere (Cook, 2006), has – as a matter of historical fact – not drawn much attention in the physics community. In 1949, the IPM (shell model) achieved the important goal of providing a quantum mechanical basis for nuclear structure theory, in a manner that was formally similar to the quantum mechanics of electron orbitals. The earlier (1937) and then much later (Lezuo, 1974;

Cook, 1976) demonstration of the curious FCC geometry of those nucleon states failed at that time to elucidate any new aspect of nuclear physics, and was consequently dismissed as a "quasi-classical analog" of the quantum mechanical reality – a numerological "coincidence" without physical implications. Subsequently, however, the substructure of the FCC lattice has been shown to have two noteworthy realms of application where the conventional models are known to be deficient. Most pointedly, since the 1960s the shell model has consistently and repeatedly predicted stable or near-stable (10^{15} years, Moller & Nix, 1994) "superheavy" nuclei at Z>112. The experimental and technological innovations in studying "superheavies" have been impressive, but the half-lives of the "superheavies" up to Z=115 are in the range of one second, with no indication of stability (Kumar, 1989). In other words, the long-range, "effective" nuclear potential-well implied by the shell model is demonstrably incorrect. The FCC lattice, on the other hand, predicts the attenuation of the periodic chart (Z~114) due to the saturation of the short-range binding among nucleons with a continued increase in coulomb repulsion (Cook, 1990).

A second area in which conventional models have been shown to be incorrect concerns the mass asymmetries of the fragments produced by the thermal fission of U^{235}. Known experimentally since 1938, the 3:2 mass asymmetry of the fission products of the actinides is frequently said by experts in fission (Vandenbosch & Huizenga, 1973; Moreau & Heyde, 1991) to be the "oldest unsolved problem" in nuclear physics. The LDM predicts ***symmetrical*** fragments; the shell model predicts fragments with "magic" numbers of protons or neutrons; and both sets of predictions are incorrect. The lattice model unambiguously predicts fragmentation along lattice planes, with a mass asymmetry of 3:2 (Cook, 1999).

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