A Mystic Dream of FOUR

Introduction

Long ago, the seeds of two theories of fundamental physics germinated in the genius of Einstein, one the seed of a quantum force theory excluding gravity, the other the seed of a non-quantum, non-force, theory of gravity. However, today the mature standard model (SM) of the quantum force, and the mature standard model of the non-quantum, non-force, are incompatible, driving modern physicists the world over to seek a SM compatible quantum force theory of gravity [1]. Out of this effort has emerged a controversial theory of motion, the superstring theory (ST) of quantum vibrations, which attempts to explain all the forces of the SM, as properties of motion, including the force of gravity [2].

Nevertheless, ST exacts a heavy price in its mathematical complexity, and even worse than that, it appears to be unverifiable, as a result of its extra dimensions and ultra small scale [3] [4] [5]. Yet, many are convinced that ST's success in explaining fundamental forces, as properties of motion, is a great step forward that cannot be easily dismissed. Indeed, the recognition that all forces are properties of motion leads to the investigation of other forms of vibration that may not require the extra dimensions and the ultra small scale of ST. One of these new approaches reformulates the four-dimensional spacetime of general relativity theory (GR), into the four-dimensional space/time of a new system of theory, called the Reciprocal System of Physical Theory (RST) [6], which leads to the development of a new quantum theory of motion, based on 3D vibrations, the vibrations of 3D balls, rather than the vibrations of 1D strings [7].

The difference is that in the new theory the vibration of these "balls" is nothing more than the relation of space and time, as two, *reciprocal*, aspects of order in progression [8]. In this manner, the observed expansion of space and time are formulated as the initial conditions of a universe of motion, where the introduction of 3D, or pseudoscalar, vibrations, at given locations in the progression, quantizes the progression at those locations. Consequently, the history of physical entities, their time lines, need no longer depend on a single event in the past, but may originate anywhere, anytime, as independent events in the growing block universe of 4D spacetime [9].

However, because these quanta are *space/time* ratios, their inverses, as *time/space* ratios, are as equally as likely to form as their dual counterparts, thereby establishing a symmetrical set of spatial and temporal pseudoscalar/scalar ratios that may represent a mathematical group, under addition. The properties of the elements of this set lead to further combinations that may be elements of a set that represent a group under multiplication.

Thus, a set of physical elements, consisting of nothing but 3D space/time and time/space oscillations, having the properties of symmetry, which is the characteristic of the laws of physical conservation [10], emerges from nothing more than the observation that scalar time is the inverse of pseudoscalar space in the order of progression. Exploiting these entities of motion to build a toy model of the structure of the physical universe, leads to (pseudoscalar/scalar) + (scalar/pseudoscalar) space and time combinations that take three forms of increasing complexity: The first form is a one-dimensional combination, identified with the bosons of the SM; The second is a two-dimensional combo, identified with the first generation fermions of the SM, and the third is a three-dimensional combo, identified with the baryon hadrons of the SM, which form the elements of the periodic table, as shown in figure 1 below.

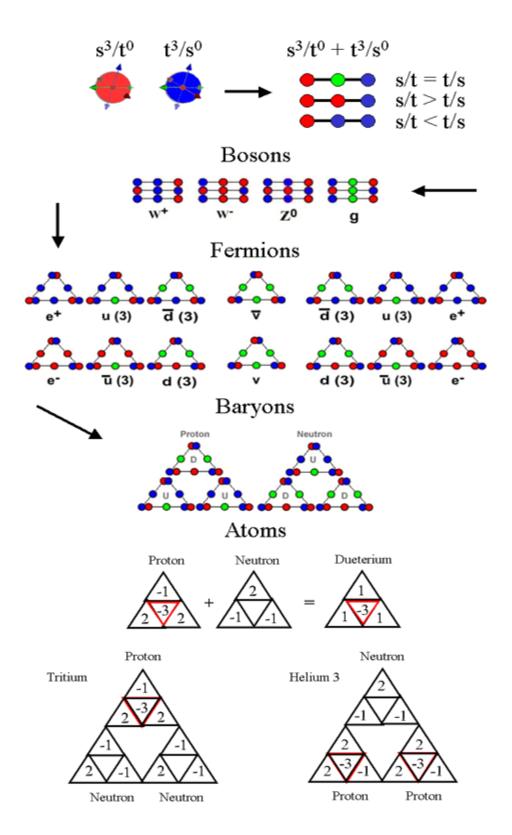


Figure 1. Toy Model showing the combinational hierarchy of 3D oscillations. Spatial (red) and temporal (blue) pseudoscalars, combined as one S|T unit, are symbolized as circles at two ends of a line, with the color of the line's middle circle (red, green or blue), indicating the relative number of each type of pseudoscalar in the combination.

In figure 1 above, we see the 3D spatial oscillations, and their inverses, combine to form combinations identified as bosons, fermions, hadrons and atoms, up to ³He. Some further isotopic combinations of the periodic table are shown in figure 2 below, up to ⁷Li, with reduced detail.

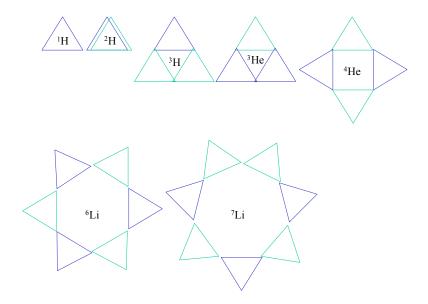


Figure 2. Periodic Combinations of Protons + Electrons (blue triangles) and Neutrons (green triangles)

Fundamental Space and Time

Of course, constructing an interesting toy model is one thing, but demonstrating its correspondence with physical observations is quite another. The tough question is, do these combinations of pseudoscalar oscillations exhibit both the mathematical and physical properties and the behavior of radiation, energy and matter? To answer this question definitively is the object of ongoing research, but some important fundamentals can be pointed out in this essay.

Currently, one of the greatest obstacles to progress in fundamental physics research is the apparent emergence of space at small scales, and, by implication, spacetime, which means that time also must be emergent [5]. Though some have attempted to construct timeless physics, based on the Wheeler DeWitt equation [11] [12], most physicists are not convinced that such a thing will ever be possible [13]. In the new system of theory discussed here, this problem is entirely avoided by recognizing that combining the physical constants G, c and h, to derive the fundamental Planck length, which is at the heart of the problem [14], may not be necessary, after all.

Instead of combining the constants of Newton (G), Einstein (c) and Planck (h), so fundamental to the SM, GR and ST, the new system takes a new approach by combining the constants of Einstein and Rydberg instead. In this way, the constant c of the expanding universe is "folded" into fundamental space and time units, by assuming that the Rydberg frequency of hydrogen, R_y , is the fundamental frequency of the expanding universe. On this basis, the rotation of frequency, $R_y = 1/t$, is treated as a unit velocity, v = c, where the space/time ratio of c = s/t, is 1/1, for the purpose of calculating the *natural* units of space and time, entering into the velocity c, thus defining the space/time magnitudes of the pseudoscalars and scalars of the 4D progression.

To calculate the natural time unit, the frequency, R_y , must be doubled; that is, since one 2π rotation per second, or $2\pi/t = 2\pi$ radians/sec, actually constitutes a unit velocity of π radians per half-second, or of 1

unit of π radians, in each half of the rotation, for every half-second, then the frequency must be doubled to determine the unit of time entering into the velocity c, as first shown in [15]. Accordingly, the inverse of $2R_y$, is $1/2R_y$, or

$$t_n = 1.5198 \times 10^{-16} \text{ seconds.}$$

Now, with a natural unit of time in hand, multiplying this unit times the constant c, gives us the corresponding fundamental unit of space,

$$s_n = c/2R_v = 4.5563 \times 10^{-6} \text{ cm}.$$

What this means, in the final analysis, is that we are able to define a multi-dimensional type of motion, differing from, but not replacing, the familiar definition of 1-dimensional motion. The familiar vector motion requires the change in an object's actual, or probable, location, for the purpose of defining a change in space over time, but under the new definition, a moving object is not required to define this change. We call this newcomer to the theoretical scene, scalar motion, a 3D, massless, motion, taking its place, by virtue of the definition of motion, along side the familiar definition of 1D vector motion, the motion of mass in one direction at a time. Since rigid objects can only move in one direction at a time, if they are to remain intact, scalar motion is not the vector motion of objects, but is simply a reciprocal relationship of order in the space/time progression, requiring no object in its equation. It exists by virtue of defining the nature of time, as the observed, 0D, temporal progression, and identifying it as the necessary physical progression that is the inverse of the spatial progression, which we identify as the observed, 3D, physical progression, producing the cosmic expansion of the universe [16].

Fundamental Theory

Interestingly enough, however, this actually goes right to the heart of the matter in the fundamental crisis of theoretical physics. The SM needs the fixed background of spacetime to describe the evolution of the wave equation, while GR introduces gravity as dynamic spacetime itself. This leads to an enigma, causing Hawking to ask, given that gravity *is* spacetime, "How can the wave function for gravity, evolve in time?"[17] This is tantamount to recognizing that there are no physical dimensions beyond the four of spacetime, to play with, in the development of physical theory. However, if we ignore this reality and introduce the dimensions of mass separately into Einstein's 4D spacetime equation, thereby warping and curving the spacetime "fabric," to generate forceless gravity, we mess up the flat spacetime needed for the SM equations of matter, running smack into the strong gravity problem, the singularity of black holes.

With the new approach, however, the warping of the "fabric" of spacetime, a la GR, by matter, is replaced by an equivalent concept. We simply reconfigure the 4D "fabric" of spacetime coordinates, into a 4D space/time progression, where the one temporal dimension is the reciprocal of the three spatial dimensions, and where certain locations of that progression are considered to be oscillating over one unit of the progression. In this way, discrete magnitudes of pseudoscalar vibrations are formed, which live in the flat 4D spacetime of special relativity (where the space component of spacetime corresponds to the product of elapsed time and velocity, the "space" of distance between objects and events).

In this new concept, gravity is a scalar motion that is a consequence of the space/time progression, which is shared by all physical entities in common, leading to the inherent outward propagation of massless radiation, and to the inherent inward motion of massive matter. Matter within a limit of proximity, aggregates, while matter, separated by great distances, outside the proximate limit, disperses. Recently, interest has been turning to the study of an analogous pseudoscalar phenomenon, exhibiting just this type of behavior in vibrating media. These oscillating pseudoscalars are called oscillons, which have just now begun to be modeled in connection with SM entities [18].

Fundamental Mathematics

However, one of the most important questions in modern theoretical physics concerns the mathematics of the SM, which is poorly understood in many ways, and the mathematics of ST, which is not formulated in terms of a fundamental symmetry principle. The big question is: "What is the ST group?"[19]. Since the answer to this question would no doubt be of great help in the study of the 1D vibrations of the 10D ST research, given the success of the SM, based on the symmetry of the three continuous groups SU(3)xSU(2)xU(1), the prospect that three similar symmetries, in discrete form, may be present in the mathematics and geometry of the 4D space/time, pseudoscalar, oscillations, is very compelling, from a unification point of view.

Happily, to characterize the mathematical relationship of 3D space and 0D time, as motion, we need only refer to the scalars and pseudoscalars of the well-known binomial expansion. In the Greek tetraktys, that ancient icon of cosmology that maps to the first four levels of the binomial expansion, we find that the bi-directional properties of all known physical dimensions are encoded as a reciprocal relationship between its scalars and pseudoscalars. Recognizing that the single, temporal, dimension of nature has no particular direction of progression (other than order of increase), and therefore increases as a, 2^0 , scalar, while the three, spatial, dimensions have two directions of progression, in each of three dimensions, or six directions in all, and therefore increases as a *set* of 2^1 , 2^2 , 2^3 , pseudoscalars, is key to consistently combining the four dimensions of space and time into one entity, a space/time, or a pseudoscalar/scalar, progression. The two, reciprocal, aspects of this progression necessarily have reciprocal dimensions, the 0 dimensions of the scalar and the 3 dimensions of the pseudoscalar.

Combining the one temporal dimension with the set of three spatial dimensions, reciprocally, in the binomial expansion, merges them together into one entity of space/time, or motion, with no adverse effect on the familiar mathematical result (ignoring the binomial coefficients for the moment), as we expand the number of dimensions of space, from zero to three, in the four levels of the tetraktys:

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First dimension (0D = 2^0), or geometric point): s^0/t^0 = 2^0/2^0 = 1/1
Second dimension (1D = 2^1), or geometric line): s^1/t^0 = 2^1/2^0 = 2/1
Third dimension (2D = 2^2), or geometric area): s^2/t^0 = (2^1*2^1)/2^0 = 4/1
Fourth dimension (3D = 2^3), or geometric volume): s^3/t^0 = (2^1*2^1*2^1)/2^0 = 8/1
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As shown in figure 3 below, in the fourth dimension of the expansion, in the so-called Euclidean three-space, the binomial coefficients of the numerator's expansion, 1, 3, 3, 1, indexing the orthogonality of each linear, or independent, space, in its 4D algebraic structure, exhibits the reflection symmetry of the four spaces and the inverse relation of the 0D scalar and the 3D pseudoscalar; that is, there are three, orthogonal, dimensions in the 1D space and three, orthogonal, dimensions in the 2D space, while in the 0D and 3D spaces, there is only one dimension each, the 0D scalar and the 3D pseudoscalar, and these are reciprocals in the sense that no direction, 2^0 , is the dual of all directions, 2^3 , in Euclidean three-space.

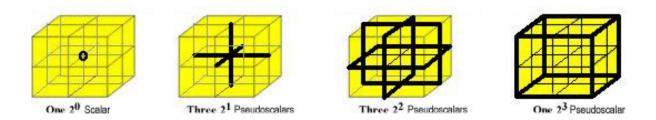


Figure 3. The Four Linear Spaces of the Fourth Geometric Dimension's Structure, Containing Eight Algebraic Dimensions

However, technically speaking, the 1D and 2D dimensions are the pseudoscalars of their respective linear spaces, as well, and may be consistently regarded as such, even though they are normally treated as coordinate dimensions of vector spaces. Consequently, while the four, independent, linear, spaces are shown separately, in figure 3, their inverse, geometric, relationship, corresponding to their inverse algebraic relationship, as one space/time entity, is more clearly illustrated in the composite image of figure 4 below.

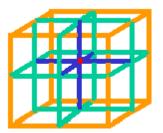


Figure 4. The Scalar (red point) as the Reciprocal of 1D (blue lines), 2D (green planes), and 3D (orange volume) Pseudoscalars

The fundamental duality, characterizing the symmetry of the space/time progression, is clear from the observation that every dimension potentially has two, opposing, degrees of freedom, expressed in the binomial expansion, 2^n , of the four linear spaces of the tetraktys, in the four dimensions of the progression. However, ever since the days of antiquity, the spaces of geometry (right lines and circles) have been at odds with the spaces of algebra, consisting of numbers of order 2^n [20]. The fundamental conflict is normally understood in the context of the square root of 2, but it's most succinctly captured in the endless attempts to "square the circle," something now proven to be impossible [21].

As it turns out, however, the unit sphere, *physically* generated by the spatial progression in all directions, fits just inside the 2x2x2 stack of unit cubes, the eightfold cube, *algebraically* generated by the unit expansion. A cross section of the unit sphere, superimposed on a cross section of the eightfold cube, is shown in figure 5 below.

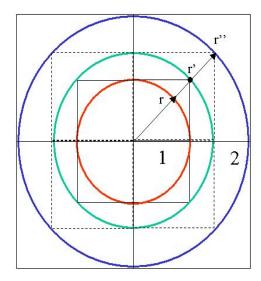


Figure 5. The Cross Section of the Eightfold Cube (Unit 3D Pseudoscalar with diagonal r'), the Real Unit Circle (red), with radius r, containing them both, and the Inverse Real Unit Circle (blue), with radius r'', circumscribing them all.

Referring to figure 5, we know that the unit sphere (red) of radius r = 1 is the inverse of the double unit sphere (blue) of radius r'' = 2, with respect to the imaginary sphere (green) of radius r', by the principles of inversive geometry [22]. Thus, by the formula for area, the area, A, of the unit sphere's cross-section (red circle), is $r^2 * \pi = \pi$, while the area, A'', of the double unit sphere's cross section (blue circle), is $r''^2 * \pi = \pi$

 4π , and they are inverses with respect to area, A', of the imaginary sphere's cross-section (green circle), which is $r'^2 * \pi = 2\pi$ (i.e. $r'^2 = r * r''$).

But, while inverses imply the duality of symmetry, and symmetry implies a group, how is 4π the inverse of 2π ? In the mathematics of GR and SM, these three radii are not treated as the basis for a group. The radius r is treated as x_4 = ict, in GR, and radius r' is treated as the unit complex number in the SM, while the radius r'' is not considered at all. Yet, it's easy to see that, relative to the area of r', the area of r is $1\pi/2\pi$, a 1:2 ratio, while the area of r'', relative to the area of r', is $4\pi/2\pi$, a 2:1 ratio, which is the inverse of the 1:2 ratio.

Fundamental Physical Symmetry

Ordinarily, we might not attach much significance to the fact that 4π is the inverse of 2π , in this sense, if it were not for the 4D progression of pseudoscalars, but it's clear that the reciprocal relation of the pseudoscalar/scalar ratio is itself symmetrical; We know that the dimensions of velocity, s/t, are just the inverse of the dimensions of energy, t/s. Thus, we can see that the reciprocal pseudoscalar/scalar relationship should apply to both velocity and energy, because one is as mathematically valid, as the other; That is, if we invert the assignment of physical dimensions, so that time is assigned the three-dimensional pseudoscalar, and space the zero-dimensional scalar, vibration in the 4D progression defines a set of temporal pseudoscalar units that are the inverses, or the duals, of the set of spatial pseudoscalar units.

Of course, if we calculate the oscillation speeds of these temporal pseudoscalars, they are supraluminal speeds, but it's not the speeds *per se* that are useful to us. It's the discrete elements of a mathematical group that are useful, and since these spatial and temporal units potentially form the elements of a mathematical group, the fact that the speed of the temporal units *appear* as supraluminal magnitudes of inverse velocity, should not concern us. Especially not, since no motion of mass is involved here, but only the relationship of the observed pseudoscalar/scalar progression (also these temporal oscillations cannot constitute tachyons, since the vibrations are temporal, not spatial).

The fundamental, discrete, temporal units, or the temporal pseudoscalars, have the same dimensions and quantities, after one unit of space progression, as do the fundamental, discrete, spatial units, or the spatial pseudoscalars, after one unit of time progression. In other words, the tetraktys, and the eightfold cube of figure 3, are invariant, regardless of which physical dimensions we assign to their elements. Of course, identifying the oscillation of temporal pseudoscalars, as the dual of spatial pseudoscalars, is one thing, but to identify it with the elements of a group, under a binary operation, is quite another thing.

How do we define a mathematical group from this set of dual space/time elements? What is the binary operation? What is the identity element? Do the elements of the set obey the requisite laws of a group under the binary operation? And, finally, most important of all, are these spatial and temporal pseudoscalars, and combinations thereof, physical representations of the group?

Obviously, proofs, and the complete answer to these questions, cannot be presented within the scope of this essay. Therefore, only a brief introduction to certain key aspects of the argument will be presented: First, it's important to understand that two interpretations of a number are possible, the quantitative and operational interpretations of number, as discussed in [20].

Second, from the perspective of increasing order, the expansion of the oscillating pseudoscalars is offset by their contraction, over two units of scalar progression, just as an increase of amplitude is offset by a corresponding decrease of amplitude in the familiar sine/cosine oscillation. Hence, there are two units of time per motion cycle, one for the expansion, and one for the contraction, or what we will call a binary oscillation, in contrast to the familiar quadrantal oscillation, described in terms of the sine and cosine of changing angle [23].

This means we can represent one cycle of the spatial pseudoscalars, per two units of temporal scalar

progression, with the space/time ratio, s/t = 1/2, and the temporal cycle with the inverse of this, s/t = 2/1. Quantitatively, the value of these two numbers equals .5 and 2, if we divide the numerators by the denominators, in the normal manner. But, operationally, the *relative value* of these rational numbers is -1 and +1, respectively, if we subtract the denominators from the numerators. The identity element of the group associated with this latter interpretation of the set of numbers, then, is obviously 0, implying that this set is isomorphic to the group of integers, under addition.

However, from a quantitative perspective, the same ratios are also quotients, where the rational number, 1/2, is the inverse of the rational number 2/1, and the identity element is 2/2 = 1/1 = 1, implying that this same set of numbers is isomorphic to the group of rationals, under multiplication. Thus, under these two interpretations of number, the single set of spatial and temporal pseudoscalar/scalar combinations appears to form a field isomorphic to that of the integers and the rational numbers combined, under addition and multiplication, respectively.

Fundamental Physics

Though proofs of these mathematical assertions need to be provided, to ensure that we can consistently add, subtract, multiply, and divide with the elements of this set of pseudoscalars/scalar combinations, it's also important to understand how these discrete entities of motion might connect to physical law, exhibiting the observed physical properties of matter, energy and radiation. All of physics is based on motion defined in terms of physical entities changing locations. Whether those locations can be measured relative to a coordinate system, with arbitrary precision or not, is beside the point here. The important measures of potential and kinetic energy, and how they play in the Hamiltonians and Lagrangians of physical theory, depend on physical variables of vector motion evolving in time, whether it's in the wave equations of the SM, or in the tensors of GR.

However, the physical dimensions of classical physics are mass, space (length) and time, not just space and time. Yet, it's been known for a long time that these can be reduced to dimensions of space and time only, as shown, for example, in [15] and [24]. Since this is the case, it follows that all the equations of classical physics should be expressible in terms of unit scalars and pseudoscalars, as space/time, time/space, ratios. Indeed, it's a matter of a trivial translation to show that some consistent form of unit correspondence exists in the equations of mass, radiation and energy, for instance:

$$\begin{split} E &= mc^2 = t^3/s^3 * s^2/t^2 = t/s \Rightarrow (t^0)^3/s^3 * s^2/(t^0)^2 = 1/2^3 * 2^2/1 = 1/8 * 4/1 = 4/8 = 1/2 \\ m &= E/c^2 = t/s * t^2/s^2 = t^3/s^3 \Rightarrow t^0/s * (t^0)^2/s^2 = (t^0)^3/s^3 = 1/2^3 = 1/8 \\ c^2 &= E/m = (t/s)/(t^3/s^3) = s^2/t^2 \Rightarrow (t^0/s)/(t^0)^3/s^3 = s^2/(t^0)^2 = 4/1 \\ E &= hv = t^2/s^2 * s/t = t/s \Rightarrow (t^0)^2/s^2 * s/t^0 = t^0/s = 1/2 \\ h &= E/v = (t/s)/(s/t) = t^2/s^2 \Rightarrow (t^0/s)/(s/t^0) = (t^0)^2/s^2 = 1/2^2 = 1/4 \\ v &= E/h = (t/s)/(t^2/s^2) = s/t \Rightarrow (t^0/s)/(t^0)^2/s^2 = s/(t^0) = 2/1 \end{split}$$

where we substitute the dimensions of velocity, for the dimensions of frequency, in the radiation equations, as already explained above.

But what, if anything, does this mean? We already know that the massless Maxwell equations are invariant in the Poincaré group [25], but can these strange massless ratios of 0D scalars and n-dimensional pseudoscalars be physically meaningful? Perhaps so, but we are so accustomed to thinking in the complex terms of units of rotation, instead of units of expansion/contraction, in the case of the SM, and in homotopic terms of diffeomorphisms, in the case of GR, that something so simple might be easily overlooked.

Yet, it's clear that if we think in terms of oscillating spheres, as analogs of rotation, in terms of changing units of π , substituting the binary oscillations of pseudoscalars for the quadrantal oscillations of imaginary numbers, new possibilities begin to materialize. These new possibilities appear to enable us to

obtain the additional degrees of freedom needed for describing charge and spin (and even isospin!) in terms of motion, without the need to resort to the obscure abstractions of complex number rotation in the Lie algebras of SU(3)xSU(2)xU(1), and without the need to resort to the extra dimensions of ST.

Nevertheless, at this point, with only a few words left in the essay's space, the details of how this might work will have to be left to subsequent essays. Certainly, we will need to investigate the space/time dimensions of the physical equations. For instance, since the time/space dimensions of mass are t^3/s^3 , it seems highly likely that the temporal component of this physical property of matter is a 3D temporal pseudoscalar, not a $(t^0)^3$ scalar, but how do we multiply/divide temporal pseudoscalars by spatial pseudoscalars and vice-versa? Hence, the paramount task is to investigate the mathematics of this set of inverse pseudoscalars, as well as the physical dimensions involved in the pertinent equations of motion.

As a final note, for what it's worth, I have to add that working with the tetraktys, the Clifford algebras, the geometry of Euclid, and the magical charm of groups, and finding them all marvelously interconnected through the space, time, dimensions and numbers of physical equations, I cannot help but grow fond of it all, and of Hamilton's sonnet, inspired by the light of his rare genius, reflecting upon this tantalizing mystery:

THE TETRACTYS

Of high Mathesis, with her charm severe,
Of line and number, was our theme; and we
Sought to behold her unborn progeny,
And thrones reserved in Truth's celestial sphere:
While views, before attained, became more clear;
And how the One of Time, of Space the Three,
Might, in the Chain of Symbol, girdled be:
And when my eager and reverted ear
Caught some faint echoes of an ancient strain,
Some shadowy outlines of old thoughts sublime,
Gently he smiled to see, revived again,
In later age, and occidental clime,
A dimly traced Pythagorean lore,
A westward floating, mystic dream of FOUR.

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