The Reality of Time and of the Direction of Time

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Setting Time Aright,
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Two Quotes

- 1) “If you want to know what really exists, take your best physical theory and look hard at the mathematics.”

  Tim Maudlin, August 27, 2011, ~ 8:30 PM

- 2) “Yes, but first think hard about the mathematical tools you are using to represent the physical world.”

  Tim Maudlin, August 28, 2011, ~ Noon

Reality and Fundamentality

- When we ask whether something exists, there are three sorts of answers possible.
- 1) No, it does not exist at all. Examples: Leprechauns and Newtonian absolute velocities.
- 2) Yes, but it is not fundamental. Examples: pianos and protons.
- 3) Yes, and it is fundamental. Examples?: Strings (according to string theory) and space-time?

Is Time Real?

- We can ask about the reality of time. The most radical position is that time is not real (Barbour?).
- Somewhat less: the space-time manifold is real (fundamental or not), but any distinction between time-like, space-like and light-like *directions* in it is not real. (bare manifold)
- Less still: the distinction is real but not fundamental. (time is real but emergent)
- Finally: the distinction is real and fundamental.

Is the Direction of Time Real?

- If we accept that time is real, we can also ask about the reality of the direction of time. Einstein might seem to suggest the direction is not real at all: “For us faithful physicists, the separation between past, present and future has only the meaning of an illusion, although a persisting one.”

- More usual: the direction is real but not fundamental. E.g., the direction “towards the future” is just the direction of increasing entropy.

- But if “entropy always (or usually) increases” is not analytic, then either the direction is determined by something else, or the direction is fundamental.

Time & Direction as Fundamental

- Newton’s view abut time was quite explicit:
- “Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external…”
- For Newton, time is a collection of linearly ordered universal *moments:* “The moment of duration is the same at Rome and at London, on Earth and on the stars, and throughout all the heavens”. I will argue that if we change the relata to *events*, Newton’s view is *vindicated* by Relativity.

Fundmentality in General Relativity

- In classical GR, it is taken as fundamental at least that space-time has the topological structure of a four-dimensional manifold. (Adding extra dimensions still usually presupposes a manifold.) If one starts from here, even the distinction between space-like and time-like directions must be secured by some other structure (e.g. a Lorentzian metric), which might be fundamental or non-fundamental.

Geometrical Structure

- A geometrical space can have different levels of structure, that form a hierarchy: metrical structure, affine structure, differentiable structure, topological structure.
- The most basic of these is the topological structure.
- I will argue that the traditional mathematical tool for describing topological structure has blinded us to how both time itself and the direction of time can be fundamental. A different tool opens up a new perspective.

Topology

- In order to organize a set of points into a space, some additional structure must be imposed on them.
- The most fundamental such structure determines facts about *continuity* in the space, including the continuity of functions from one space to another. This is the *topological structure*.
- This level of structure is defined without regard to either *distance* (metrical structure) or *straightness* (affine structure): hence the rubric *rubber sheet geometry*.

Standard Topology

- The basic notion in the usual formulations of topology is the *open set*.
- Every other notion–closed set, connected space, continuous function, boundary, compactness, Hausdorff, etc.– is ultimately defined in terms of the open set structure.

The Architecture of Topology

Path
(image of a curve)

Curve (continuous function
from real line into space)

Connected
Space

Continuous
Function

Boundary of
a Set

Open set $\Leftrightarrow$ Closed set $\Leftrightarrow$ Neighborhoods

Informal Explication

- “an open set is one in which every point has some breathing space” M. Crossley, *Essential Topology*
- “In topology and related fields of mathematics, a set *U* is called **open** if, intuitively speaking, you can ‘wiggle’ or ‘change’ any point *x* in *U* by a small amount in any direction and still be inside *U*. In other words, *x* is surrounded only by elements of *U*; it can’t be on the edge of *U*.”–Wikipedia

The Axioms

- Definition: A **topological space** is a set, *X*, together with a collection of subsets of *X*, called “open” sets, which satisfy the following rules:
- T1. The set *X* itself is “open”.
- T2. The empty set is “open”.
- T3. Arbitrary unions of “open” sets are “open”.
- T4. Finite intersections of “open” sets are “open”.

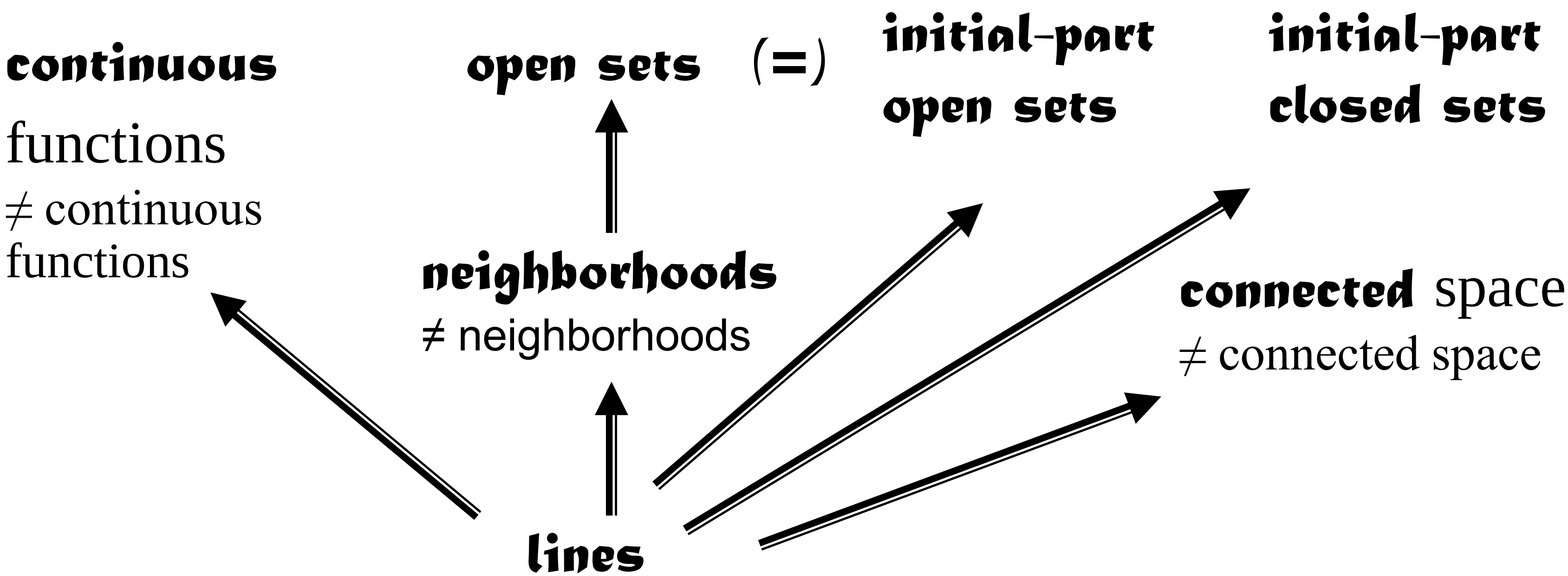
Why Should This Work?

- If this particular mathematical tool—the analysis of the continuity properties of a space in terms of its open set structure—is a direct way to describe *physical* space or space-time, then there should be some *physical* feature of the world that determines which sets of events are open sets.
- It is not obvious what such a physical feature would be. We could, of course, postulate it as a primitive fact about sets of events—that some, but not others constitute open sets—but that should be a last resort.

Alternative Geometrical Primitive: the Line

- Rather than the open set, there is a better fundamental notion upon which a theory of sub-metrical geometry can be built: the line.
- More exactly, the “open” line, in the sense that both open and closed line segments are “open” and a circle is “closed”: from any point on the line one can move continuously to any other given point, but only by moving in one direction.
- An open line in this sense has a structure represented by a linear order among the points.

Theory of Linear Structures



Linear Orders

- A linear order on a set S is a relation, which we will symbolize by "$\geq$", that satisfies three conditions:
- For all p, q, $r \in S$
- 1) If $p \geq q$ and $q \geq p$, then $p = q$ (Antisymmetry)
- 2) If $p \geq q$ and $q \geq r$, then $p \geq r$ (Transitivity)
- 3) $p \geq q$ or $q \geq p$ (Totality)

Intervals

- An *interval* in set with a linear order is a subset of at least two points such that for any *p*, *q* in the subset, all points between *p* and *q* in the order are in the subset. (Dedekind)

Linear Structures (1st type)

- A Linear Structure is a set S together with Λ a set of subsets of S called the **lines** of S that satisfy:
- LS_1 (Minimality Axiom): Each **line** contains at least two points.
- LS_2 (Segment Axiom): Every **line** λ admits of a linear order among its points such that a subset of λ is itself a **line** if and only if it is an interval of that linear order.

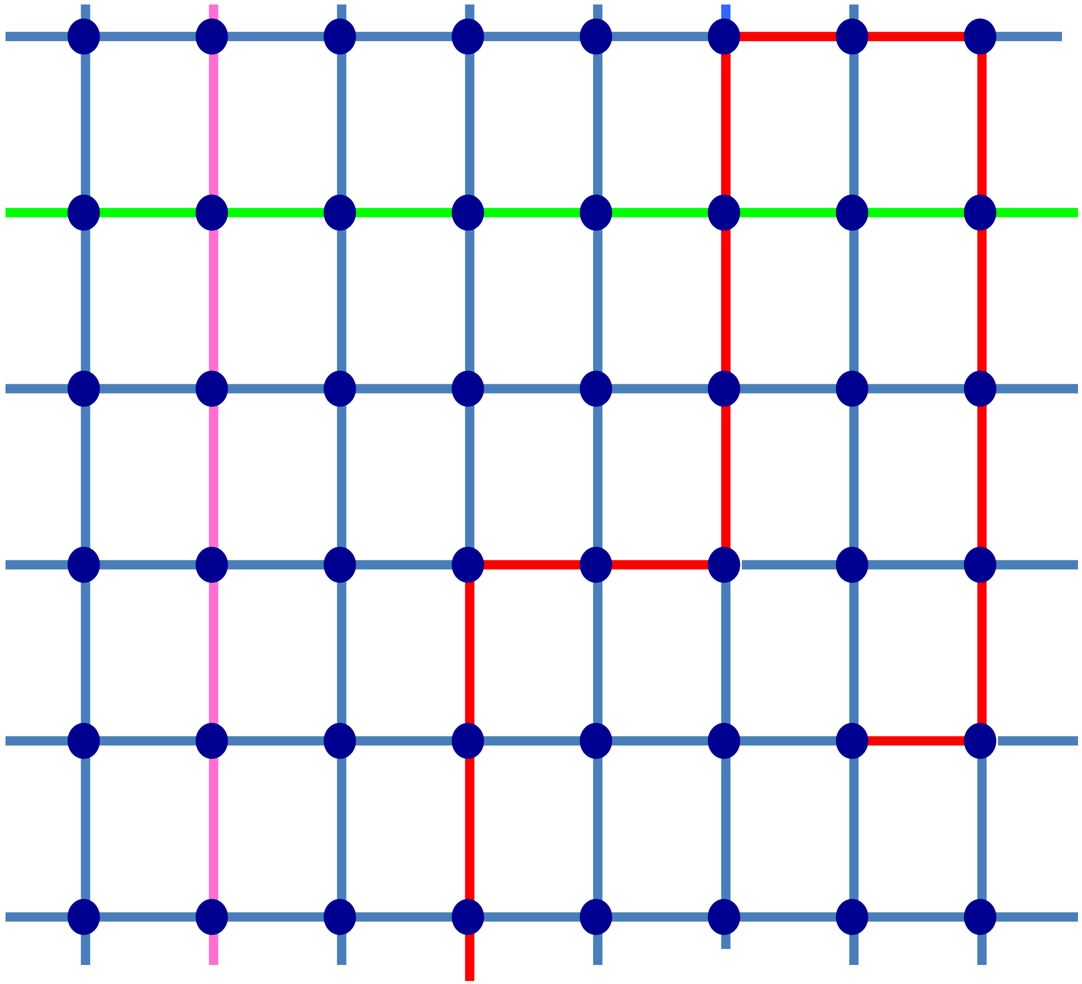
Linear Structures con't

- LS_3 (Point-Splicing Axiom): If λ and μ are **lines** that have in common only a single point p that is an **endpoint** of both, then $\lambda \cup \mu$ is a **line** provided that no **lines** in the set $(\lambda \cup \mu) - p$ have a point in λ and a point in μ.
- LS_4 (Completion Axiom): Any linearly ordered set σ such that all and only the closed intervals in the order are **closed lines** is a **line**.

Non-Uniqueness of Order

- According to this first set of axioms, every line can be represented by a linear order among its points. But evidently there are two such linear orders that will do the job, one the inverse of the other. Each will imply the same intervals, and so the same structure of **segments**. (A **segment** of a **line** λ is a subset of λ that is a **line**.)

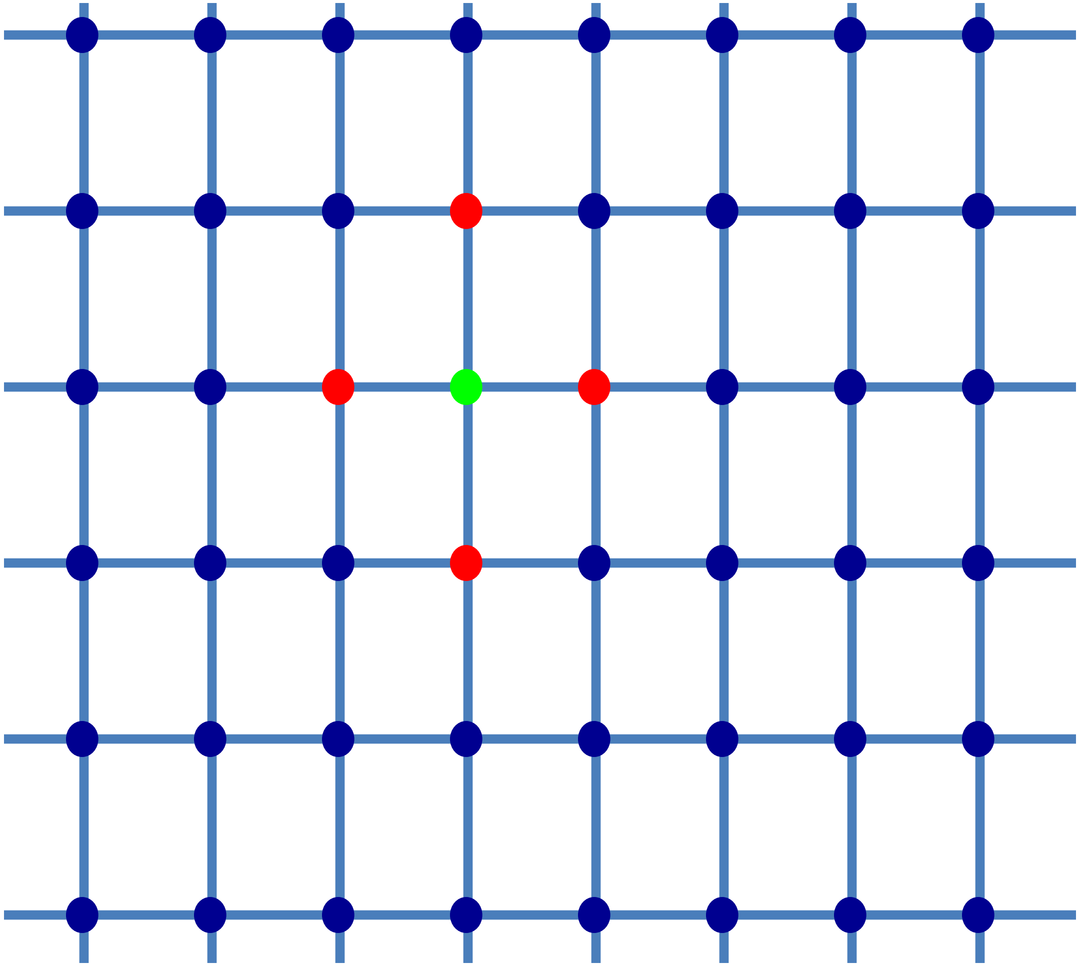
Lines on a Square Lattice



Neighborhoods

- A set Σ is a **neighborhood** of a point *p* iff every **line** with *p* as an **endpoint** has a **segment** with *p* as an **endpoint** in Σ.

Neighborhoods on a Square Lattice



Open Sets

- A set Σ in a Linear Structure is an **open set** iff it is a **neighborhood** of all of its members.
- (NB: this definition looks *identical* to a definition that appears in standard topology, with **neighborhood** replaced by neighborhood. But in standard topology, a neighborhood of a point is a set containing an *open set* containing the point.)

Theorem

- The collection of **open sets** in a Linear Structure satisfy the axioms of standard topology, i.e., the **open sets** are open sets.

Some numbers

# of points	topologies	Linear structures	Topologies from LS
1	1	1	1
2	4	2	2
3	29	8	5
4	355	64	15
5	6,942	1,024	52

This suggests….

- Evidently many (in an obvious sense most) standard topologies on a finite point set cannot be generated from a Linear Structure on that set.
- I call such topologies *geometrically uninterpretable*.

For Example

- Consider a space with only 2 points, p and q. There are four standard topologies:
- The discrete topology: $\{p, q\}, \{p\}, \{q\}, \varnothing$.
- The indiscrete topology: $\{p, q\}, \varnothing$.
- Two Sierpinski spaces: $\{p, q\}$, $\{p\}, \varnothing$ and $\{p, q\}, \{q\}, \varnothing$.

Only 2 Linear Structures

Discrete topology: open sets

$\{p,q\}$, $\{p\}$, $\{q\}$, $\varnothing$

Indiscrete topology: open sets

$\{p,q\}$, $\varnothing$

But…

- A little further thought shows this to be incorrect! We *can* understand all finite point topologies in terms of “wiggles”.
- In the sort of Linear Structure we have constructed so far, we have treated the **lines** as two-way streets: if a small “wiggle” along a line can take you from *p* to *q*, then a small wiggle along the same line can take you from *q* to *p.*

However…

- Suppose we treat the **lines** as *one-way* streets: to specify a **line** one has to specify both a set of points that constitute it *and a direction*, i.e., only *one* linear order represents a line, not two.
- The intuitive notion of a “small wiggle” is a continuous motion long a **line** in the direction of the **line**.

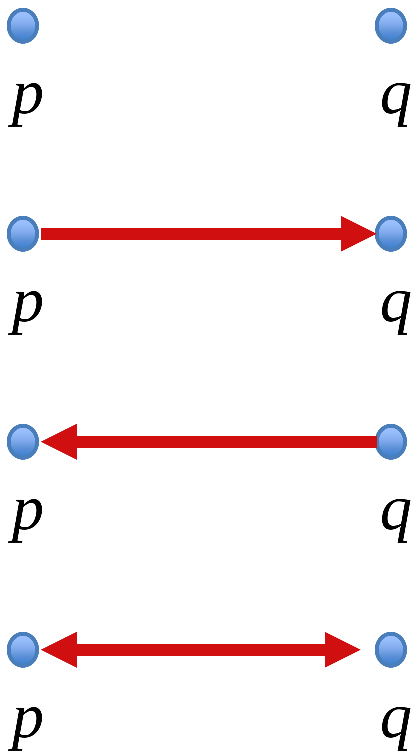
Directed Linear Structures

- This gives rise to the notion of a *Directed Linear Structure*. The axioms are modified in the obvious way: all and only the directed intervals in a linear order are **segments** of a **line**, etc.
- The Splicing Axiom now requires that to splice two **lines**, the point *p* must be the *final* point of one and the *initial* point of the other.

Outward Neighborhoods, Outward Open Sets

- A set Σ is an **outward neighborhood** of a point *p* iff every **line** with *p* as an **initial endpoint** has a **segment** with *p* as an **initial endpoint** in Σ.
- A set Σ in a Linear Structure is an **outward open set** iff it is an **outward neighborhood** of all of its members.

Directed LS for Two-Point Space



Outward open sets

$\{p,q\}, \{p\}, \{q\}, \varnothing$

$\{p,q\}, \{q\}, \varnothing$

$\{p,q\}, \{p\}, \varnothing$

$\{p,q\}, \varnothing$

More Numbers

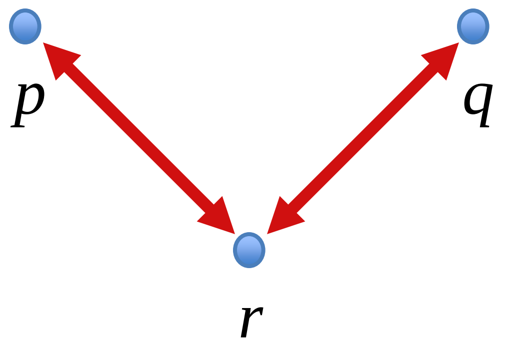
# of points	topologies	Directed LS	Topologies from DLS
1	1	1	1
2	4	4	4
3	29	64	29
4	355	4,096	355
5	6,942	1,048,576	6,942

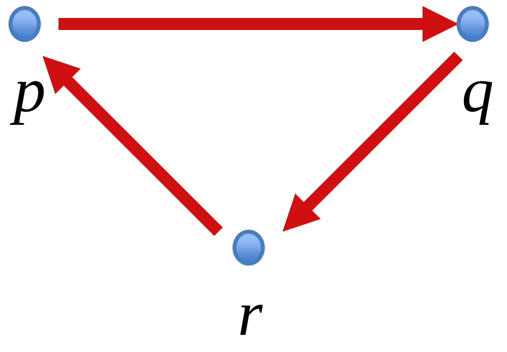
Theorem

- Every finite-point topology is generated by some finite-point Directed Linear Structure. Typically, many distinct Linear Structures give rise to the same topology, so one loses geometrical information if one only knows the topology.

Example

These DLSs generate the same topology (viz. the indiscrete topology).





Geometrically Uninterpretable Topologies

- There are, however, still geometrically uninterpretable topologies, topologies generated by no Directed Linear Structure. They all contain infinitely many points.

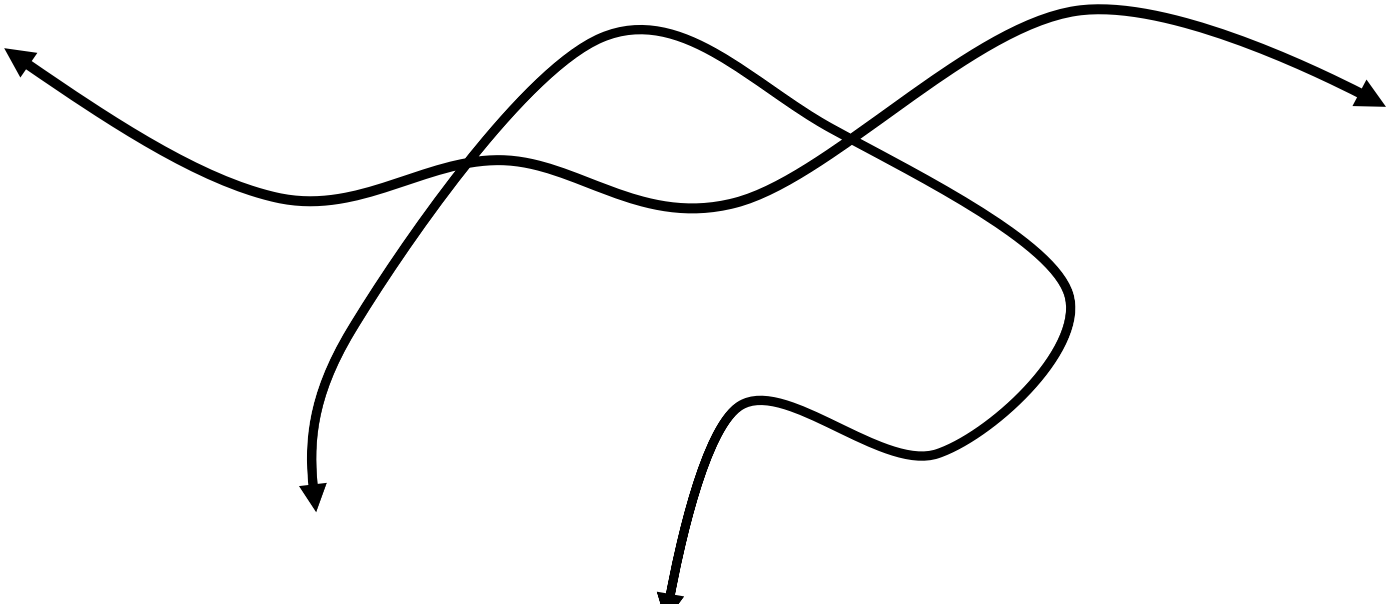
The Geometry of a Part of a Space

- In the Theory of Linear Structures, unlike standard topology, the geometry of a part of a space is defined in the natural way: by simple restriction. That is, the Linear Structure of a part of a space is given by the **lines** that are contained in that part.

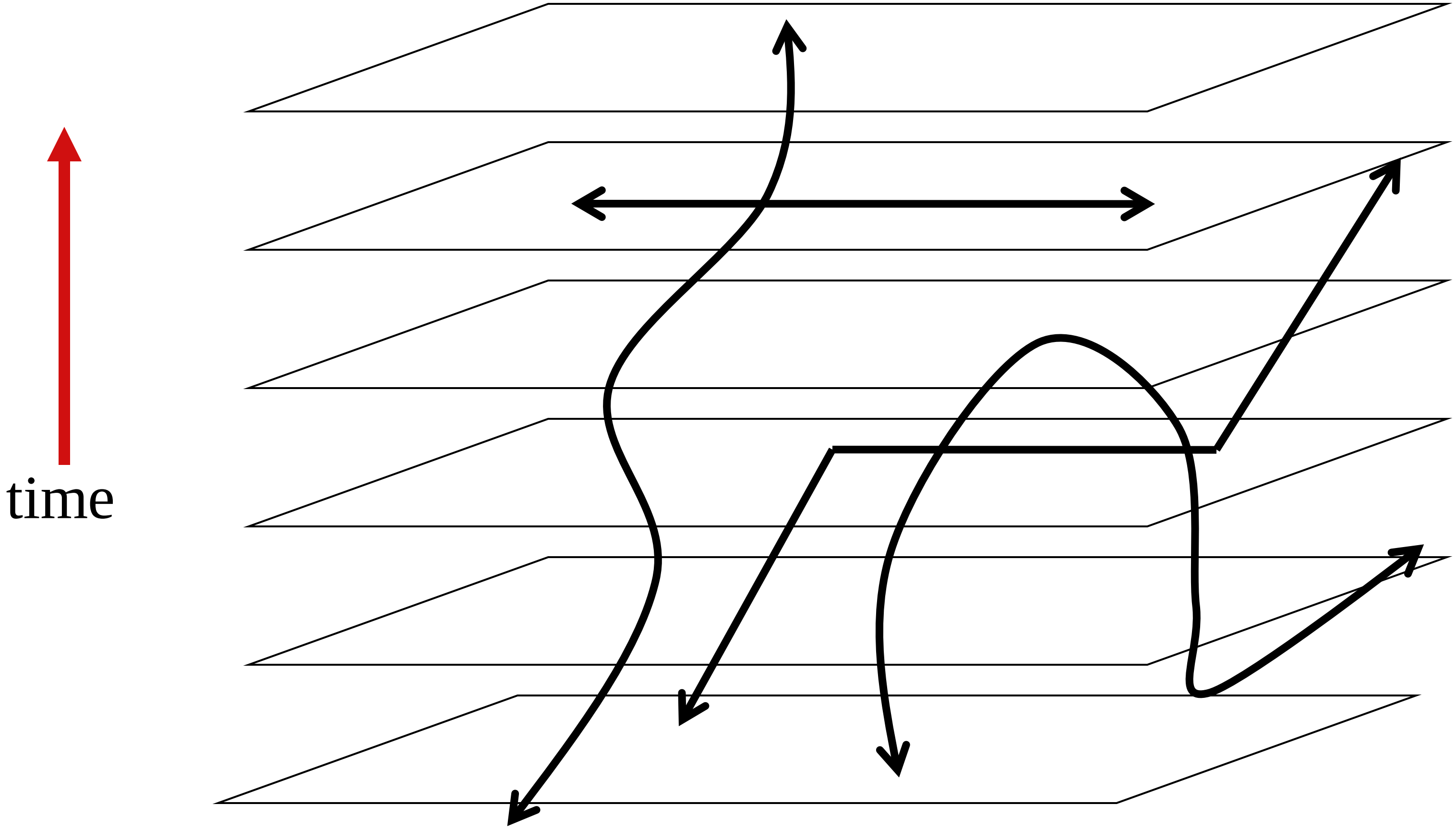
Space-Time: Why a 4-d Manifold is Unmotivated

- A topological 4-dimensional manifold has an open set structure that everywhere is locally isomorphic to a 4-d Euclidean space. From our point of view, the obvious reason to expect this would be because the Linear Structure of space-time is locally isomorphic to the Linear Structure of 4-d Euclidean space.

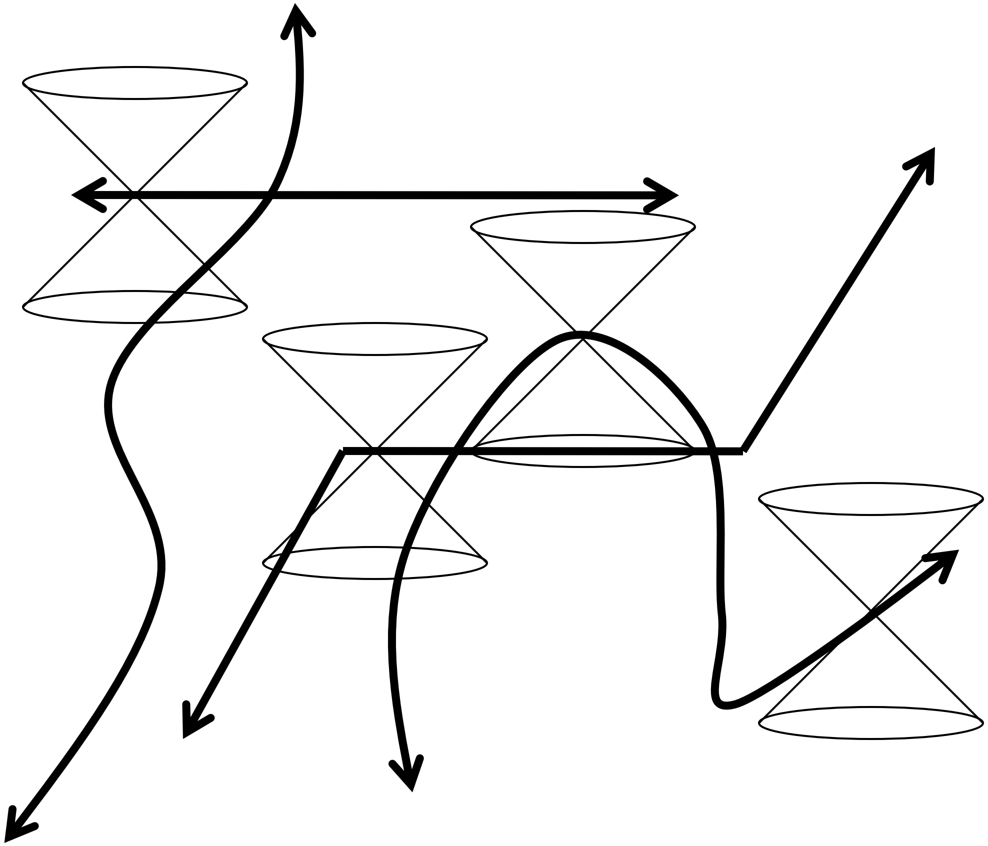
Lines in Euclidean Space



Lines in Newtonian Space-Time?



Lines in Relativistic Space-Time?



Wald on "Mixed" *Lines*

- "The length of curves which change from timelike to spacelike is not defined" (*General Relativity*, p. 44).
- So let's eliminate those curves: the Linear Structure of a Relativistic space-time is not the same as that of any Euclidean space.

Physics

- If the fundamental sub-metrical geometrical structure is the **line**, then when we turn to physics, we should ask: what *physical feature* of the universe could generate physical **lines**?
- More generally, what physical feature of the universe naturally generates a linear order among the points of space-time?

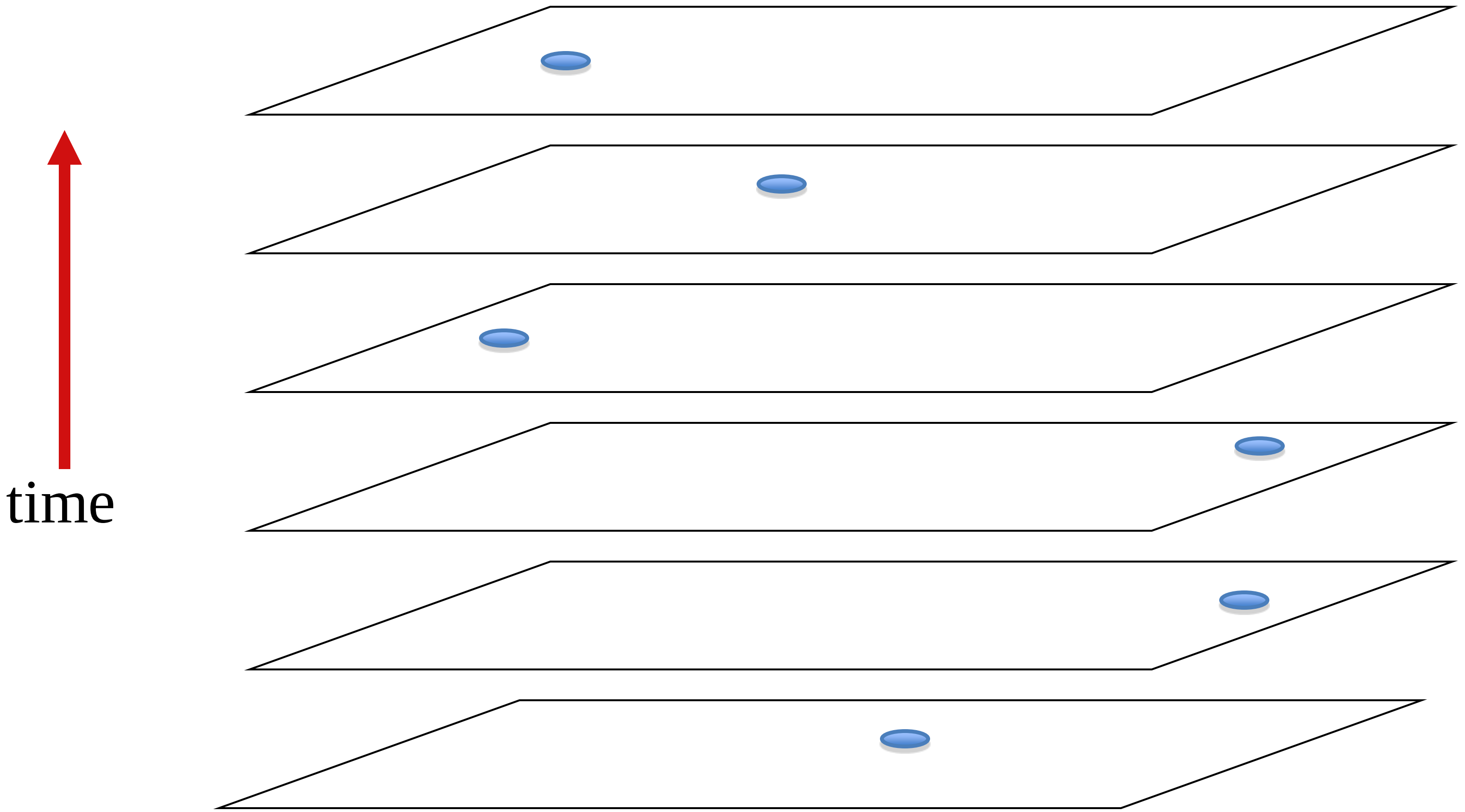
Time

- Intuitively, time provides a directed linear ordering of events. It is the natural place to look for a source of physical lines.

Newtonian and Neo-Newtonian Space-Time

- In Newtonian or Neo-Newtonian space-time, if one asks for a *maximal set of events which is linearly ordered in time*, one gets a set of points, one at each instant of time. This set of points will not typically look like any sort of line:

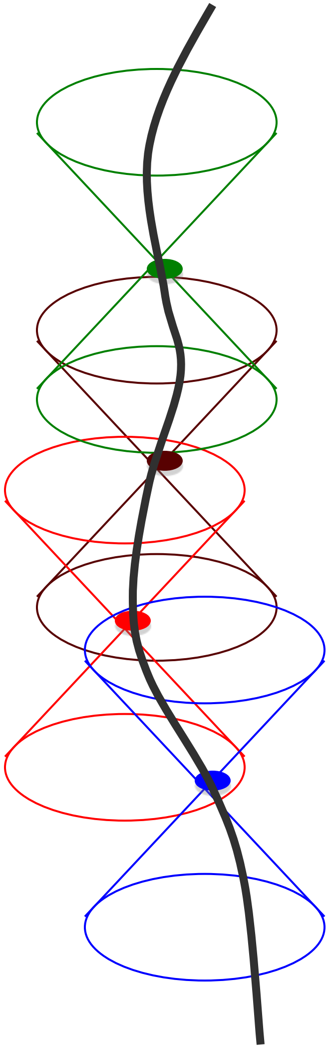
Newtonian Space-Time



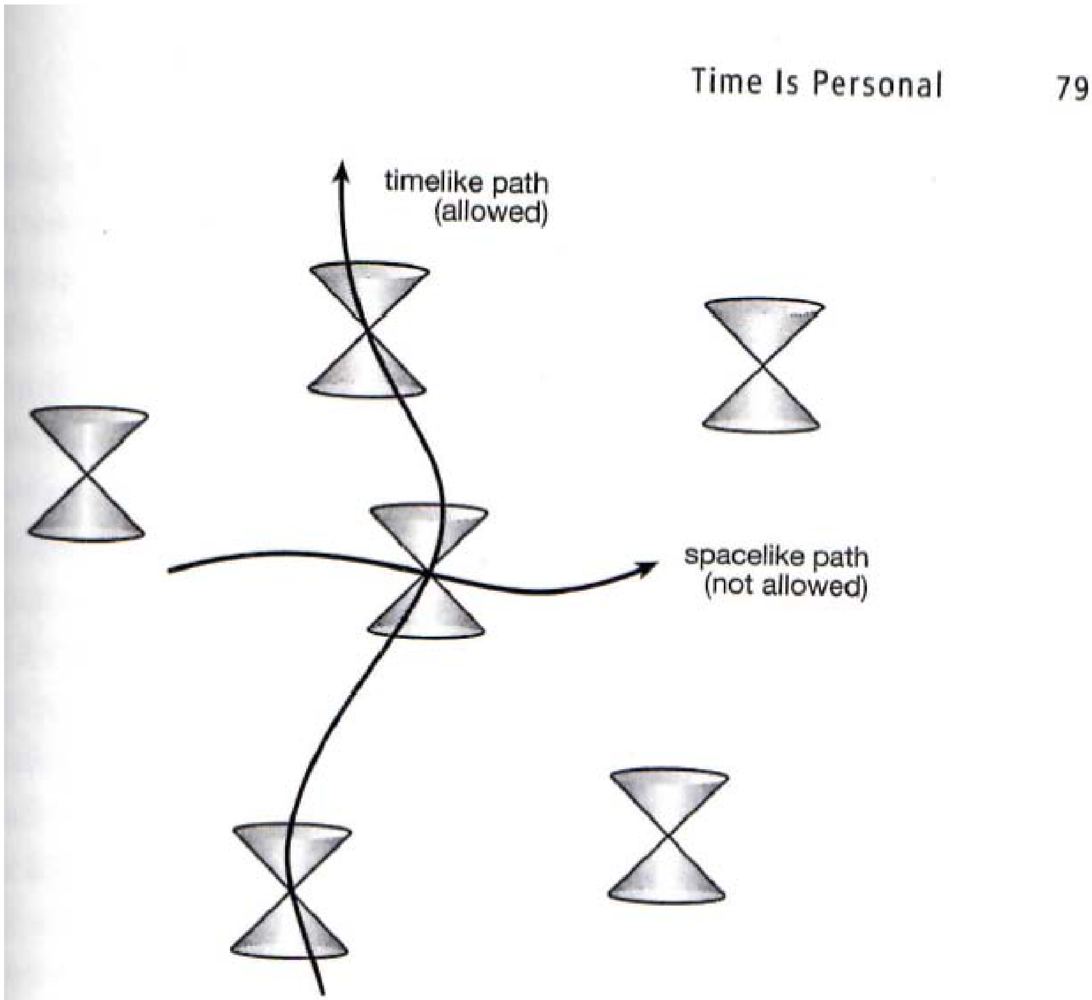
Relativistic Space-Time (Globally Hyperbolic)

- In a Relativistic space-time (Lorentzian pseudo-metric) with no closed time-like curves (no “time travel”), if one asks for a *maximal set of events which is linearly ordered in time*, what one gets is a *continuous time-like or null curve*.
- The light-cone structure forces such a set to intuitively form a line.

Relativistic Space-Time



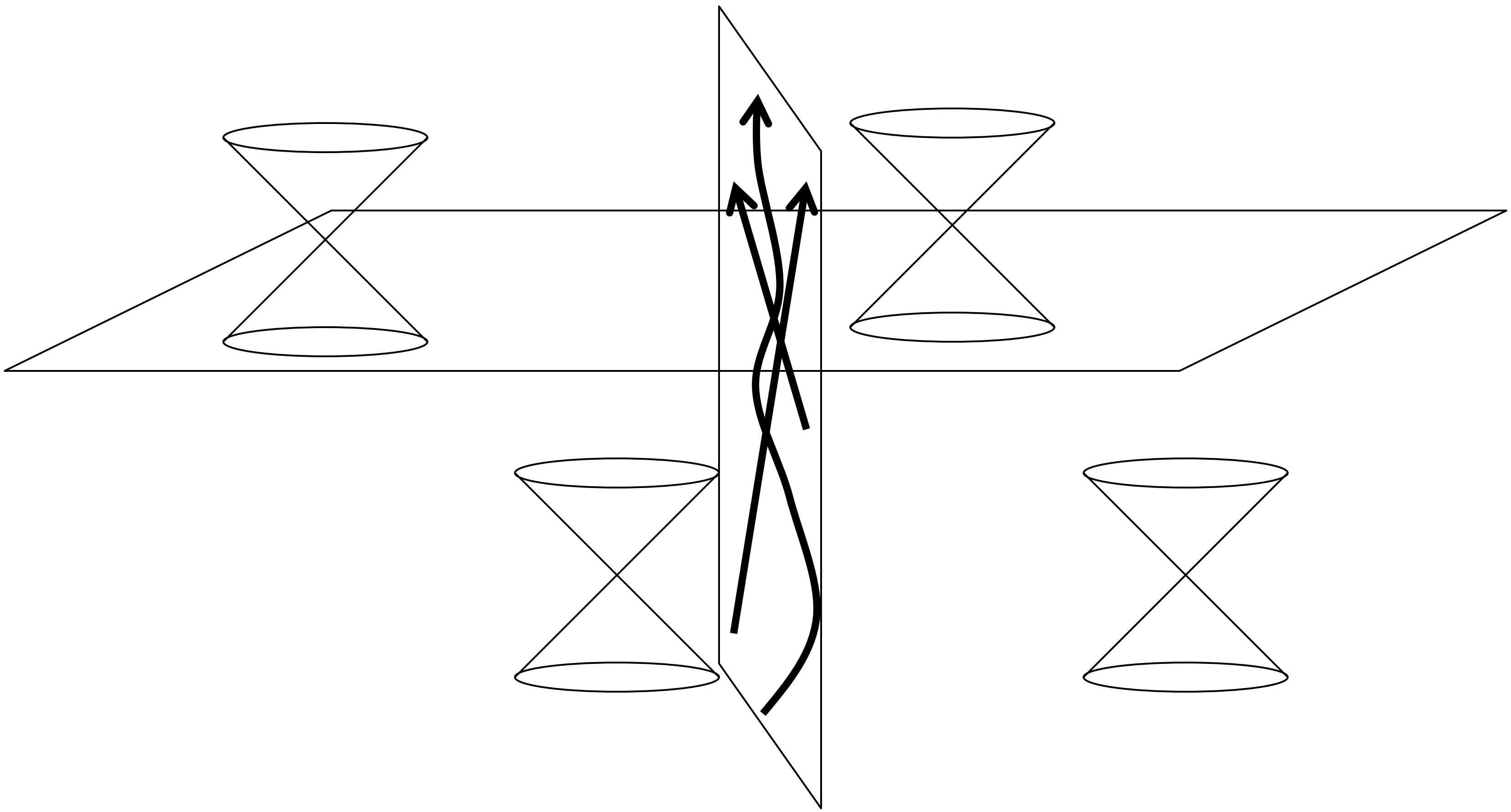
From Sean Carroll's Book



Only Trivial Geometry on a Spacelike Hypersurface

- If we only admit timelike-or-null **lines**, then when we restrict the geometry to a space-like hypersurface we get no **lines** at all: there is no intrinsic spatial geometry.
- Other slices, though, have the expected Relativistic structure.

Linear Structure of Hyperplanes



Relativistic Structure is Built Into the Linear Structure

- If we follow this recipe, the light-cone structure of a space-time is already definable from its Linear Structure, without use of any metrical notions.
- In particular, a **closed line** with **endpoints** *p* and *q* is a **straight lightlike line** just in case it is the only **closed line** with these endpoints. So one can recover the light-cone structure directly from the Directed Linear Structure.

Recovering the Whole Relativistic Metric

- To get the full Relativistic (pseudo-)metric, one needs to attribute a “length” to these lines, i.e. the proper time along them. This is enough to determine all the spatio-temporal structure postulated by Relativity.

How the Mathematics Describes Physics

- If we use the Theory of Linear Structures to characterize the geometry of a space, then the topology is determined by the **directed lines**—linearly ordered sets of points—in the space.
- If we accept that time linearly orders events, then the maximal sets of temporally ordered events form a natural *physical* **directed linear structure** in space-time.
- In Relativity—but not classical physics—this turns out to be just the geometrical structure the physics need. Time invests space-time with this geometrical structure.