Gravity as Machian Shape Dynamics

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Alternative Title:

Was Spacetime a Glorious Historical Accident?

Time will not be fused with space but emerge from the timeless shape dynamics of space. Absolute simultaneity restored!

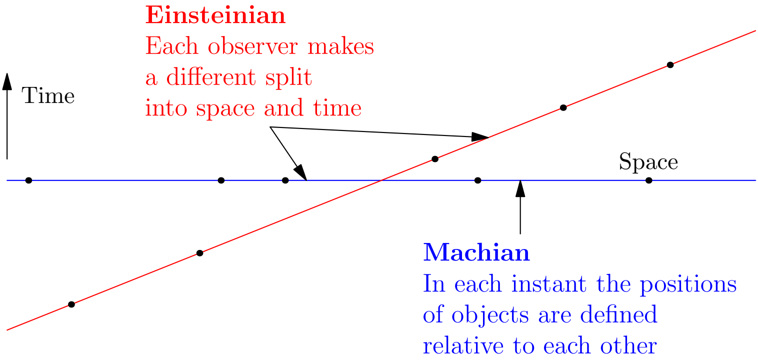
Overview

How Is Motion to be Defined?

Three possibilities:

1. By a Newtonian-type absolute background of space and time.
2. By boundary conditions at spatial infinity (e.g. Schwarzschild).
3. **By Machian dynamics in a closed universe.**

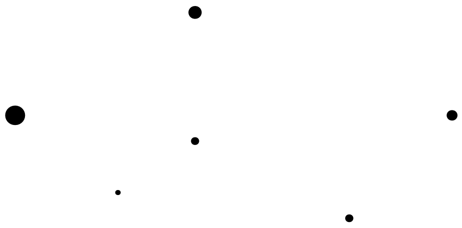
The Two Notions of Relativity



Systematic Machian relativity casts doubt on Einsteinian relativity.

Dirac, 1958: "I am inclined to believe that four-dimensional symmetry is not a fundamental property of the physical world."

Relative Configurations as Shapes and Instants of Time



The relative configurations, or **shapes**, of the Universe do not occur at instants of time $t \in R$. They are the instants of time.

The Universe is like bees 'swarming in nothing'. How do we describe that?

Was die Welt im Innersten zusammenhält

"He that attempts natural philosophy without geometry is lost."

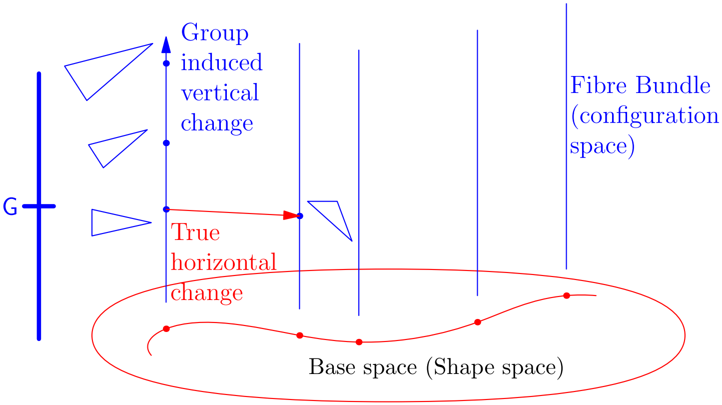
The algebraic relations connecting the $N(N-1)/2$ distances r_{ab} between N points $\Longrightarrow$ embeddability in Euclidean space. Spatial geometry "holds shapes together".

The Different Representations of Empirical Facts:

1. By empirical measured distances (carry their own semantics).
2. By Cartesian coordinates (redundant, need interpretation).
3. By "true" degrees of freedom (hard, also need interpretation).

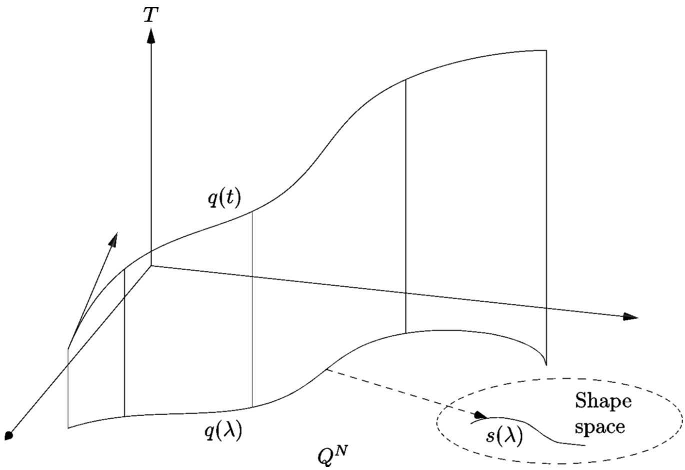
The relations between the r_{ab} suggest vector spaces, groups, transformations of coordinates, and fibre bundles.

The Three-Particle Shape Space as Fibre Bundle



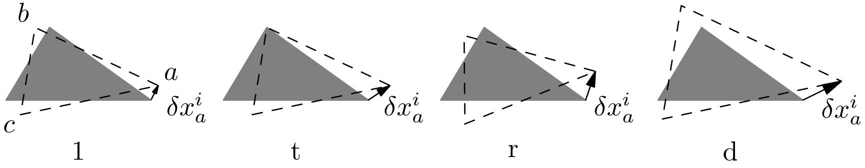
The *structure group* G generates fictitious vertical change. Nature generates real horizontal change. Defining it is *the central problem.*

Elimination of Newton's Unobservable Background



λ is a mere label of shapes. There is no time metric.

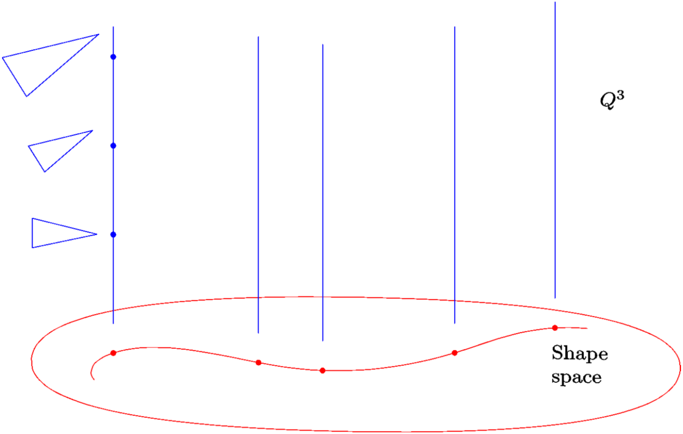
The Mach/Poincaré Principle



Rotations (r) and dilatations (t) relative to the initial placing (1) do not change the observable initial data in Shape Space but do change the subsequent evolution. By Galilean invariance, translations (t) have no effect.

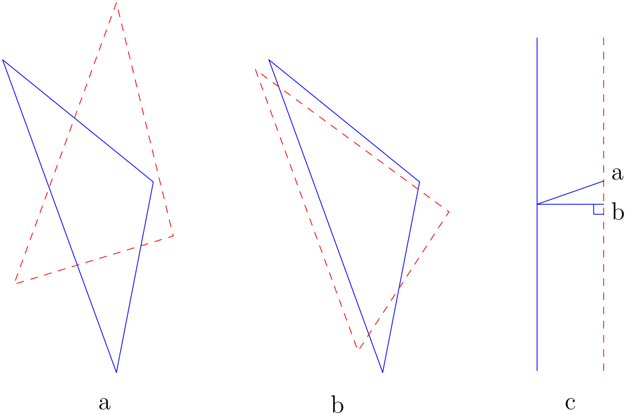
The Mach/Poincaré Principle: A point and a tangent vector must determine the evolution uniquely.

Implemention of the Mach/Poincaré Principle



Define a metric and hence geodesic principle on Shape Space using the canonically defined metric on the fibre bundle.

Creation of Metric on Shape Space by Best Matching



Arbitrary (a) and best-matched (b) placings of dashed triangle.

$\delta s_{bm} :=$min of $\sqrt{\frac{W}{I_{cms}} \sum_a \frac{m_a}{2} \delta \mathbf{x}_a \cdot \delta \mathbf{x}_a}$ between orbits

The Linear and Quadratic Constraints

The best-matched momenta $\mathbf{p}_a^{bm} := m_a \frac{\mathrm{d}\mathbf{x}_a^{bm}}{\mathbf{d}\lambda}$ satisfy

$$\mathbf{P}^{bm} := \sum_a \mathbf{p}_a^{bm} = 0 \quad \text{(translational bm)}$$

$$\mathbf{L}^{bm} := \sum_a \mathbf{x}_a^{bm} \times \mathbf{p}_a^{bm} = 0 \quad \text{(rotational bm)}$$

$$D^{bm} := \sum_a \mathbf{x}_a^{bm} \cdot \mathbf{p}_a^{bm} = 0 \quad \text{(dilatational bm)}$$

$$\sum_a \frac{m_a}{2} \mathbf{p}_a^{bm} \cdot \mathbf{p}_a^{bm} = W/I_{cms} \quad \text{(quadratic geodesic constraint)}$$

The Definition of Motion

"The Universe is given only once, with its relative motions alone determinable" (Ernst Mach, 1883).

Enlargement of structure group diminishes true motions:

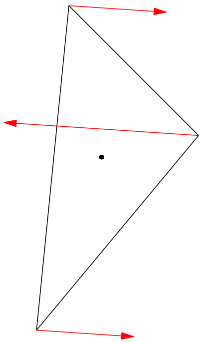
$$\mathbf{P}^{bm} := \sum_a \mathbf{p}_a^{bm} = 0 \quad \text{(translational bm)}$$

$$\mathbf{L}^{bm} := \sum_a \mathbf{x}_a^{bm} \times \mathbf{p}_a^{bm} = 0 \quad \text{(rotational bm)}$$

$$D^{bm} := \sum_a \mathbf{x}_a^{bm} \cdot \mathbf{p}_a^{bm} = 0 \quad \text{(dilatational bm)}$$

Choice of structure group converts Mach's intuition into theory.

Pictorial Representation of Constraints

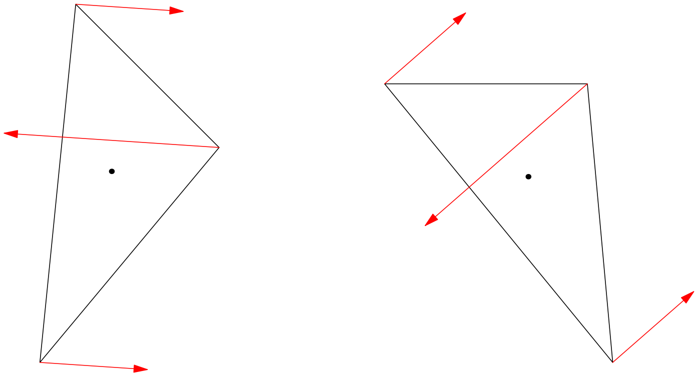


The hedgehog's momentum spikes satisfy linear constraints.

The quadratic constraint adds conditions on spike lengths.

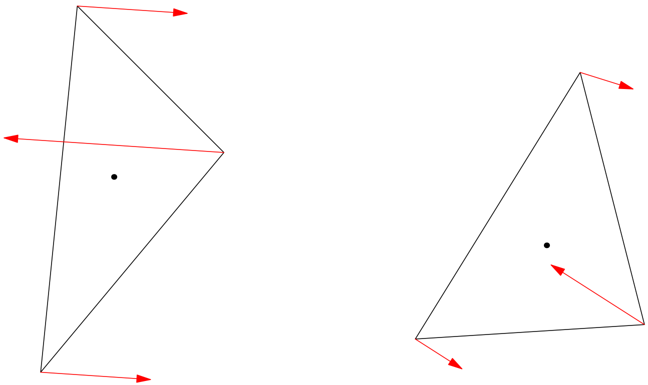
Both kinds of constraint have similar effect on hedgehog shape.

Effect of Linear Constraints



Linear constraints map q, p hedgehog to a similar one.

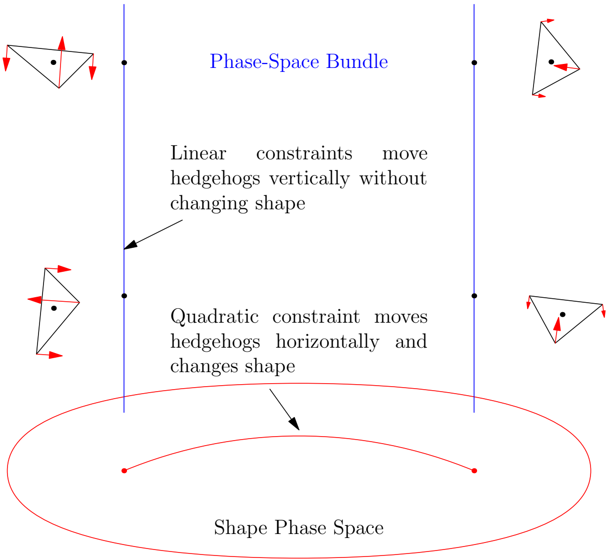
Effect of Quadratic Constraint



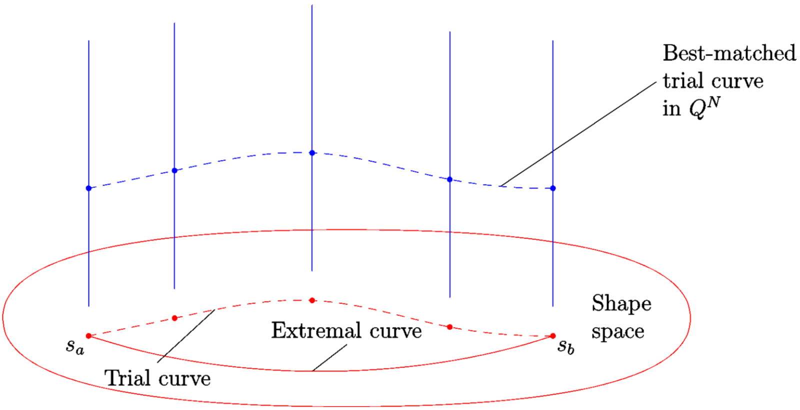
Quadratic constraint maps q, p hedgehog to a dissimilar one.

Quadratic constraint generates *undeniable* change.

The Phase-Space Bundle



The Two-Stage Variational Procedure



Emergent Time from Best-Matched Dynamics

$$A_J = 2\int \mathrm{d}\lambda \sqrt{(E-V)T}, \;\; T = \sum_a \frac{m_a}{2} \frac{\mathrm{d}\mathbf{x}_a^{bm}}{\mathrm{d}\lambda} \cdot \frac{\mathrm{d}\mathbf{x}_a^{bm}}{\mathrm{d}\lambda}$$

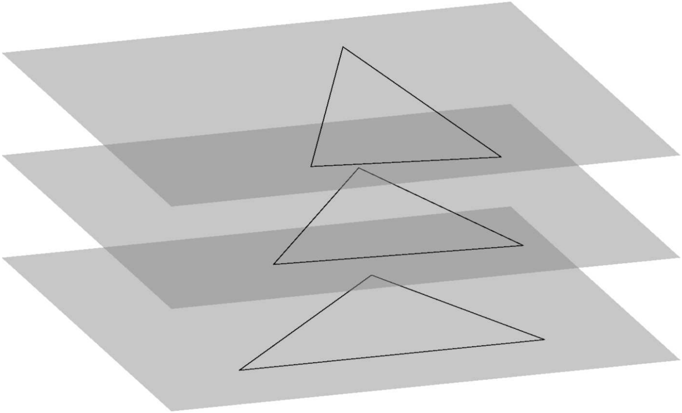
$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\sqrt{\frac{(E-V)}{T}} m_a \frac{\mathrm{d}\mathbf{x}_a^{bm}}{\mathrm{d}\lambda}\right) = -\sqrt{\frac{T}{(E-V)}} \frac{\partial V}{\partial \mathbf{x}_a^{bm}}$$

Simplify by choosing λ such that always $T = E - V$. Then

$$\frac{\mathrm{d}^2\mathbf{x}_a^{bm}}{\mathrm{d}t^2} = -\frac{\partial V}{\partial \mathbf{x}_a}, \quad \mathrm{d}t := \sqrt{\frac{\sum_a m_a \mathrm{d}\mathbf{x}_a^{bm} \cdot \mathrm{d}\mathbf{x}_a^{bm}}{2(E-V)}}.$$

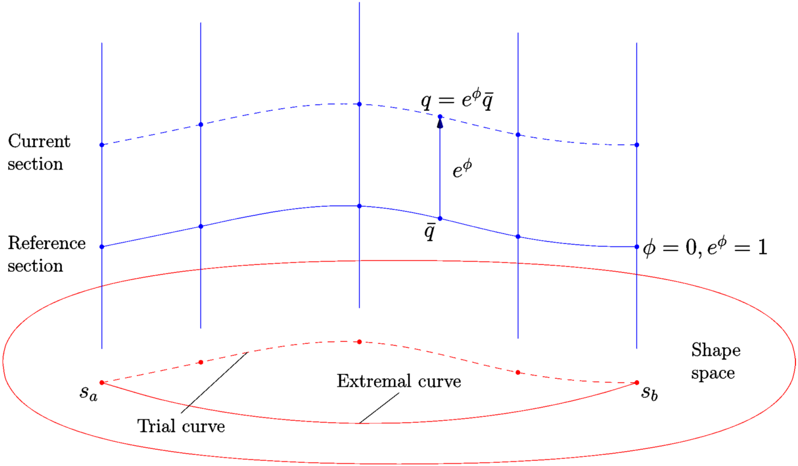
"It is utterly impossible to measure the changes of things by time. Quite the contrary, time is an abstraction at which we arrive from the changes of things." (Ernst Mach, 1883) Doubly holistic!

The Construction of Newtonian Spacetime



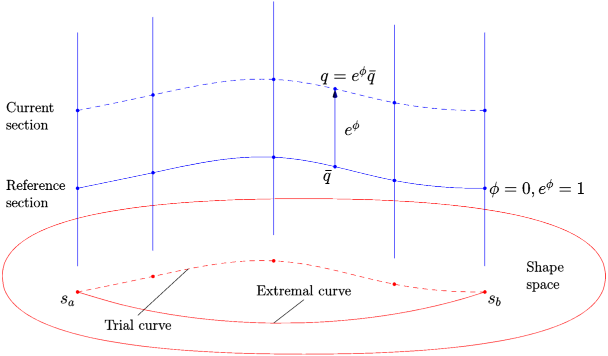
Best matching effects horizontal stacking. Emergent time fixes vertical separations. This *distinguished representation* does not change physics, just simplifies the equations. It's 'what we feel'.

Bundle Coordinates



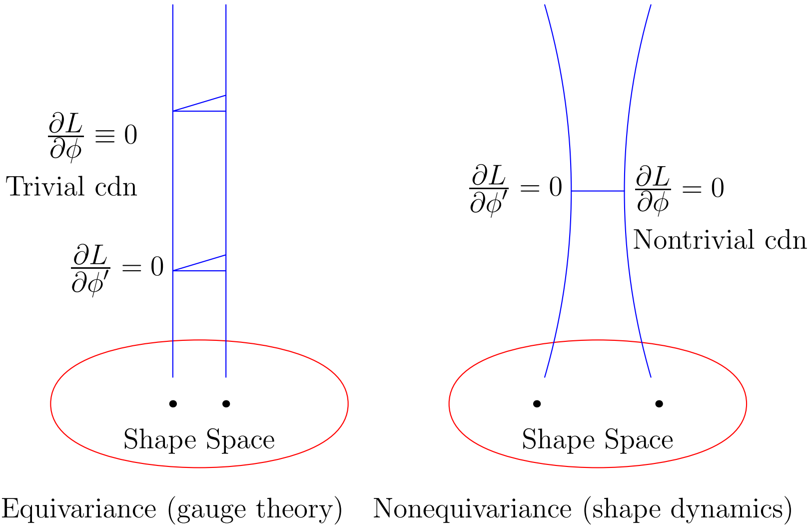
Gauge invariance: $q \to e^{\omega} q = e^{\omega+\phi}\bar{q}$, $\phi \to \phi - \omega$ leaves current section unchanged.

Formal Theory of Best Matching



1. **Coordinatize bundle:** $q = e^{\phi}\bar{q}$, **define** $\int \mathrm{d}\lambda\, L(q, q'), q' = \frac{\mathrm{d}q}{\mathrm{d}\lambda}$.
2. **Mach variation of** $L^*(\bar{q}, \bar{q}', \phi, \phi') := L(q, q')$: $\frac{\partial L}{\partial \phi'} = 0, \ \frac{\partial L}{\partial \phi} = 0$.

Equivariant and Nonequivariant Actions



Riemannian Three-Metrics

A Riemannian 3-metric $g_{ij}(x)$ defines dimensionful distances:

$$\mathrm{d}s = \sqrt{g_{ij}\mathrm{d}x^i\mathrm{d}x^j}$$

and dimensionless cosines of angles:

$$\cos\,\theta = \frac{g_{ij}\mathrm{d}x^i\mathrm{d}y^j}{\sqrt{g_{kl}\mathrm{d}x^k\mathrm{d}x^l}\sqrt{g_{mn}\mathrm{d}y^m\mathrm{d}y^n}}$$

The six cpts g_{ij} contain coordinate (3), scale (1) and angle (2) information. Angles invariant under conformal transformations

$$g_{ij} \to \phi^4 g_{ij},\ ({}^3R \to \bar{R} = \phi^{-4}R - 8\phi^{-5}\nabla^2\phi).$$

In shape dynamics only angles physical.

Geometrodynamic Configuration Spaces

Riem (M):= Space of all g_{ij} defined on the CWB 3-manifold M.

$$\text{Superspace} := \frac{\text{Riem}}{\text{3-diffeomorphisms}}$$

$$\text{Conformal Superspace+Volume (CS+V)} := \frac{\text{Superspace}}{\text{VPCTs}}$$

$$\text{Conformal Superspace (CS)} := \frac{\text{Superspace}}{\text{conformal transformations}}$$

VPCTs are *volume-preserving* conformal transformations, which also leave angles invariant. Base space for shape dynamics is CS+V.

Volume-Preserving Conformal Transformations

Full conformal transformations:

$$g_{ij} \to e^{4\phi} g_{ij}, \ \phi > 0.$$

Volume-preserving conformal transformations (VPCTs):

$$g_{ij} \to e^{4\hat{\phi}} g_{ij}, \ \hat{\phi} = \phi \frac{\int \mathrm{d}^3 x \sqrt{g}}{\int \mathrm{d}^3 x \sqrt{g} \phi^6}, \ \phi > 0.$$

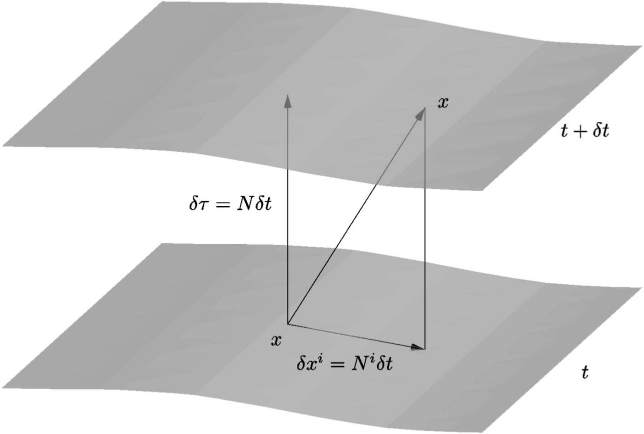
Crucially removes a single global degree of freedom.

Constant conformal transformations:

$$g_{ij} \to e^{4\phi} g_{ij}, \ \phi = \mathrm{const} \ > 0.$$

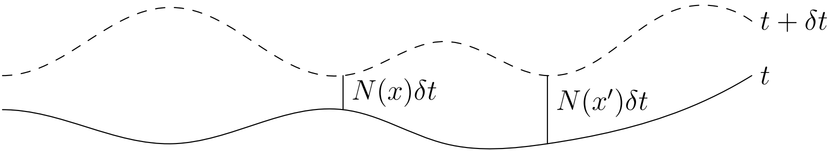
These either 'expand the universe' or change the unit of distance.

The ADM 3+1 Decomposition



The ADM dynamical variables are g_{ij} and π^{ij}.

Many-Fingered Time



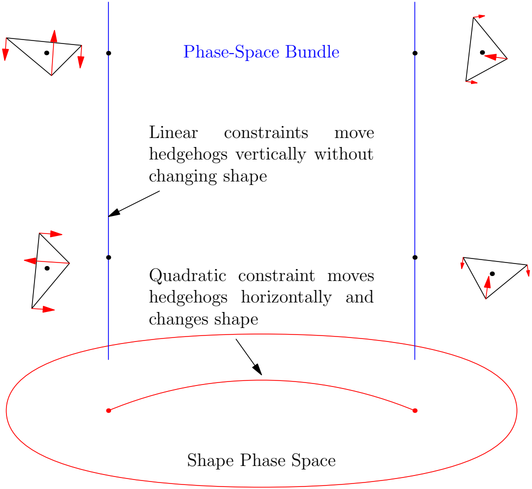
The 'hedgehog-changing' ADM quadratic Hamiltonian constraint:

$$-\pi^{ij}\pi_{ij} + \pi^2/2 + R = 0, \ \pi = g_{ij}\pi^{ij}$$

The ADM linear best-matching momentum constraint

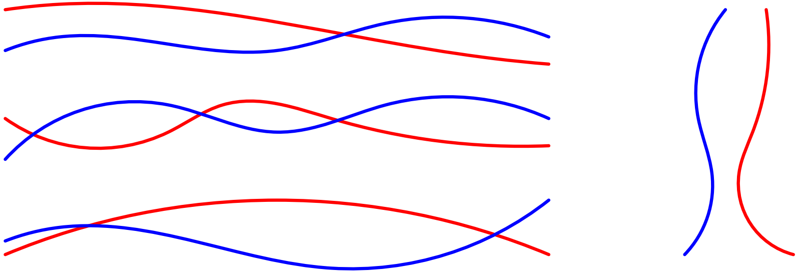
$$\pi^{ij}{}_{;j} = 0$$

Once More the Phase-Space Bundle



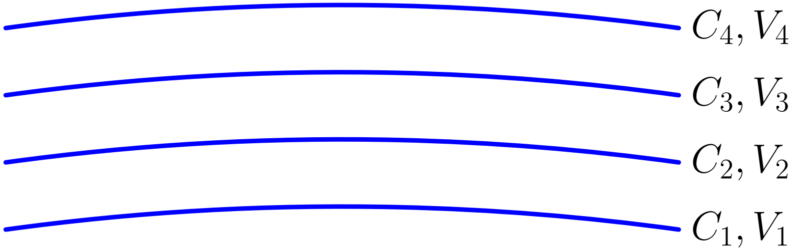
The ‘hedgehogs’ now represent the 3-geometry and its momenta.

The Effect of Foliation Invariance



Instead of a unique geodesic in Superspace, there is a 'sheaf of geodesics'.

Constant-Mean-Curvature(CMC) Foliations

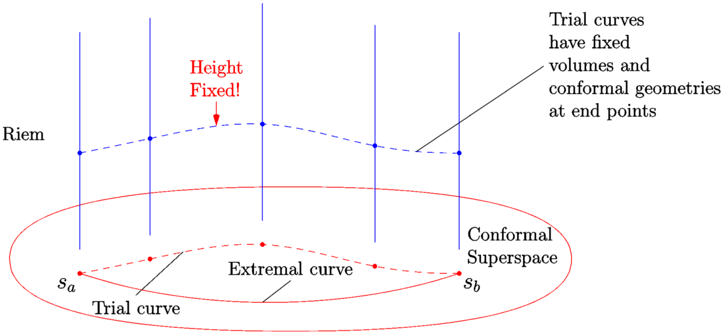


Analogous to soap bubbles in three dimensions.

Used by York to solve initial-value problem of GR.

The mean extrinsic curvature C increases monotonically.

The Bare Shape-Dynamic Action Principle



$$A_{SD} = \int \mathrm{d}\lambda \int \mathrm{d}^3x \sqrt{gRT}, \; g = \det\, g_{ij}, \;\; T = G^{ijkl} g'_{ij} g'_{kl}$$

$$G^{ijkl} = g^{ik} g^{jl} - g^{ij} g^{kl}$$

Very natural but why local square root and $-g^{ij}g^{kl}$? Height in bundle fixed by nonequivariance. Unique curve in Superspace.

The Constraints of Shape Dynamics

$$\pi^{ij} := \frac{\delta A_{SD}}{\delta \dot{g}_{ij}}, \quad \pi = g_{ij}\pi^{ij}, \quad \sigma^{ij} := \pi^{ij} - \frac{1}{3}g^{ij}\pi$$

Diffeomorphism constraint: $\pi^{ij}{}_{;j} = 0$.

Conformal constraint: $\frac{\pi}{\sqrt{g}} = C(\lambda)$ (spatial constant).

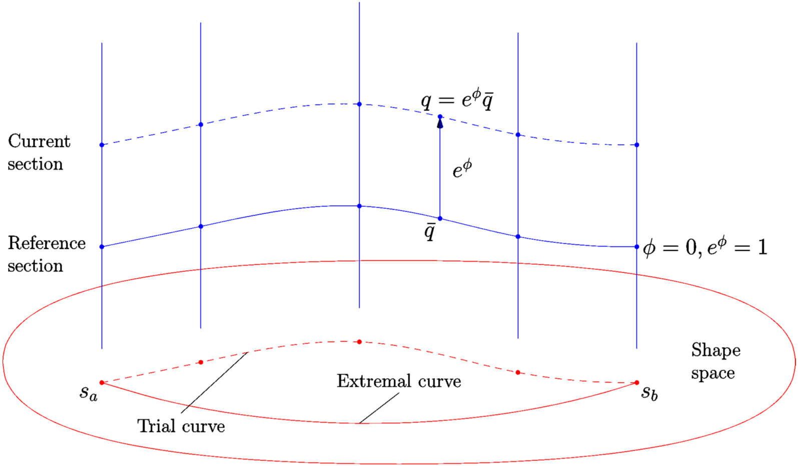
Quadratic constraint leads to

Lichnerowicz–York eqn: $\sigma^{ij}\sigma_{ij} - \frac{\pi^2\hat{\phi}^{12}}{6} - g\hat{\phi}^8\left(R - 8\frac{\nabla^2\hat{\phi}}{\hat{\phi}}\right) = 0.$

Consistency condition $\delta A_{SD}/\delta\phi = 0$ leads to

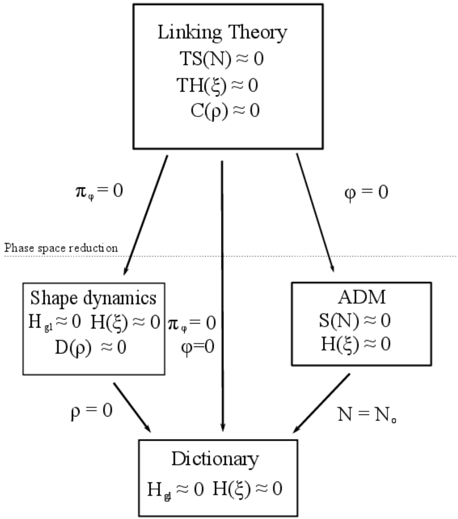
Lapse fixing cdn: $NR - \nabla^2 + \frac{NC^2}{4} = D$ (spatial constant)

Transition to Distinguished Representation

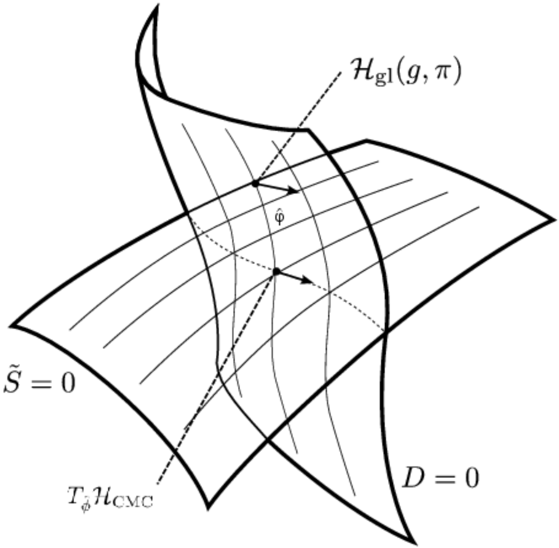


Use gauge invariance $q \to e^{\omega} q = e^{\omega+\phi}\bar{q}$, $\phi \to \phi - \omega$ to move reference section to current section. Then ϕ disappears.

The Linking Theory of Gomes, Gryb and Koslowski



The Two Gauge Theories on One Phase Space



Summary

Specification of a point and tangent vector in conformal superspace (CS) determines a slab of spacetime in CMC foliation and unique curve in CS.

Almost perfect implementation of Mach's principle because local inertial frames, local proper distance and local proper time all emergent and determined by the universe's shape and shape velocity.

The Mystery: Shape velocity, as opposed to shape direction, is last vestige of Newton's absolute space and time. Responsible for expansion of the universe and perhaps perfect transformation theory in quantum theory of the universe.